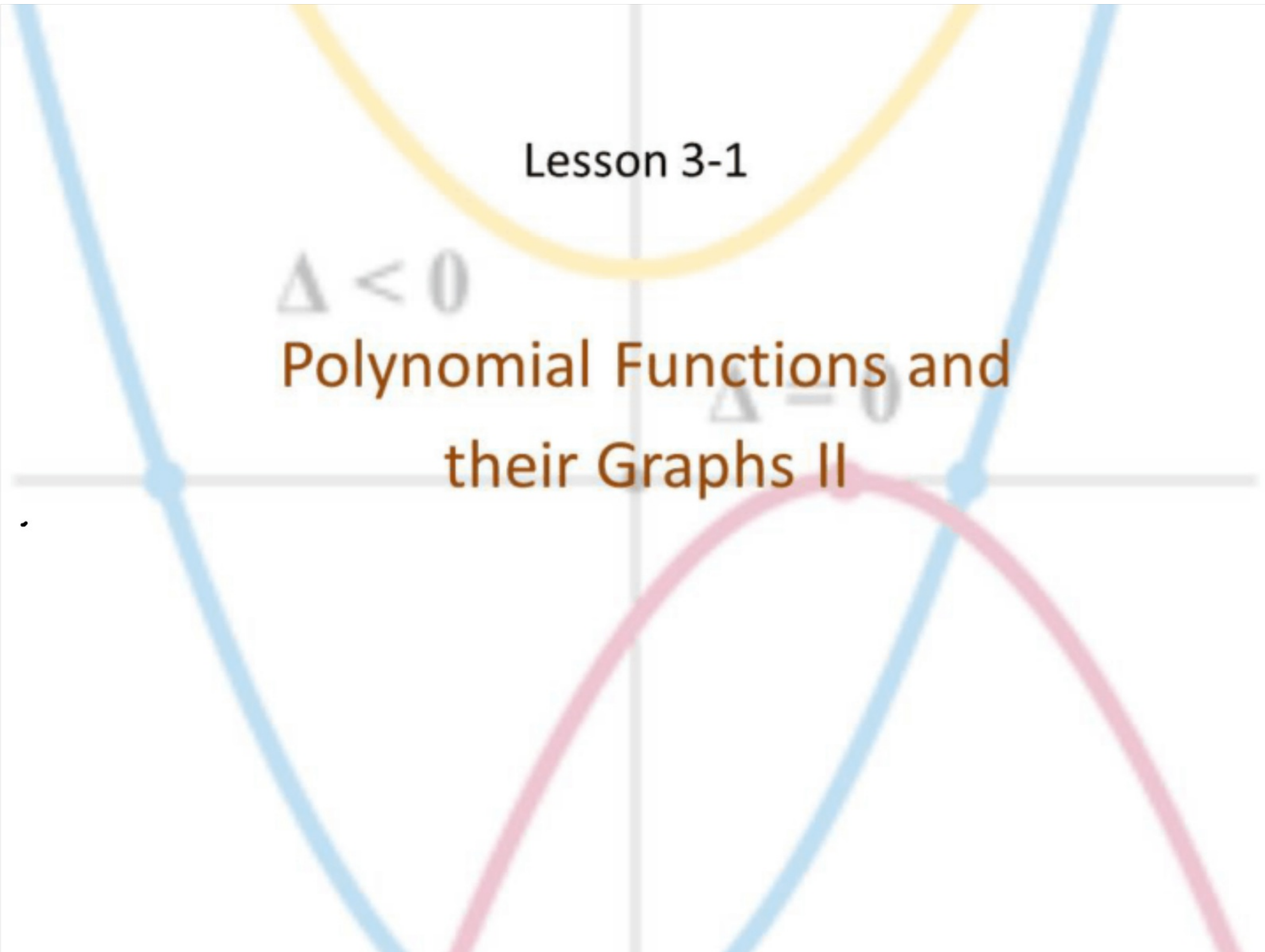

Lesson 3-1
Δ < 0
Polynomial Functions and
their Graphs II

Objective

Students will…

- Be able to find the zeros and describe the end behaviors of polynomials.

Polynomial Functions

A polynomial function consists of polynomials, which take the form

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_1 x^1 + a_0 x^0,$$

where *n* is a nonnegative integer and $a_n \neq 0$.

The numbers $a_1, a_2, \ldots, a_n$ are called the coefficients.

The number a_0 is the constant coefficient or constant term.

The number a_n, the coefficient of the highest power, is the leading coefficient, and the term $a_n x^n$ is the leading term.

Graphs of Polynomials

We already know that the graphs of polynomials of degree 0 or 1 are lines, and the graphs of polynomials of degree 2 are parabolas. The pattern is that the greater the degree of polynomials, the more complicated its graph can be.

However, there are two distinct natures of every polynomial graphs.

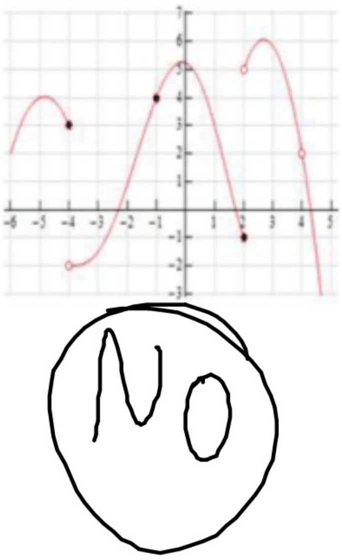
- Polynomial graphs are always smooth and curvy, i.e. no sharp corners

- Polynomial graphs are continuous, i.e. no breaks in between.

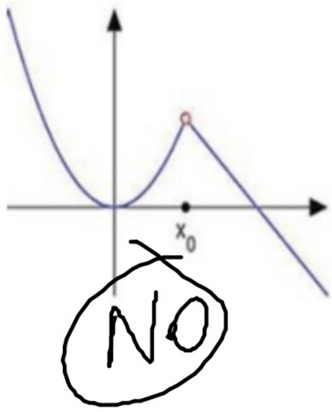
Example

Determine whether following graphs are polynomial functions.

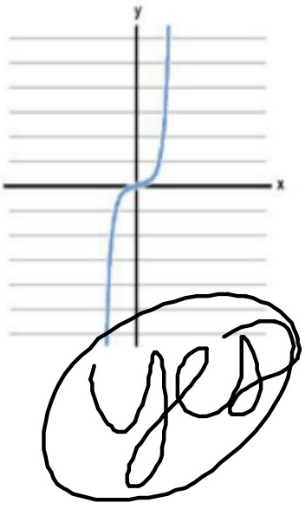
1.



2.

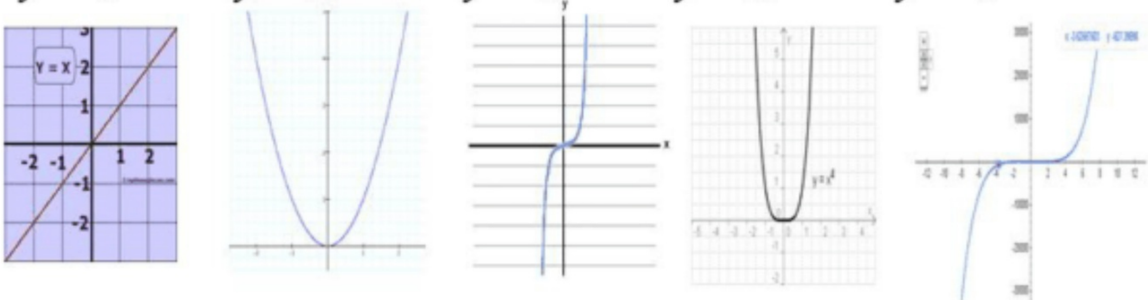


3.



The simplest polynomial functions are the monomials $P(x) = x^n$, whose graphs are often referred to as the standard graph. We have already seen standard graphs of linear and quadratics.

$y = x$ $\quad$ $y = x^2$ $\quad$ $y = x^3$ $\quad$ $y = x^4$ $\quad$ $y = x^5$



As you can see, as degree becomes larger, the graphs become flatter around origin and steeper elsewhere. We can see that all the odd number degree graphs are a variation of $y = x$, while all the even number degree graphs are a variation of $y = x^2$.

End Behavior

We saw a few distinct natures and patterns involving polynomial graphs. There is also a pattern regarding their end behavior.

Notation:

$x \to \infty$ means "x becomes large in the positive direction"

''As x goes to infinity.'',

Right.

$x \to -\infty$ means "x becomes large in the negative direction",

Left

$y \to \infty$ means "y becomes large in the positive direction"

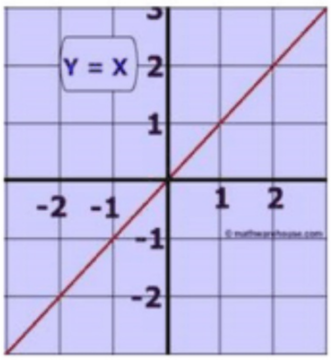
UP

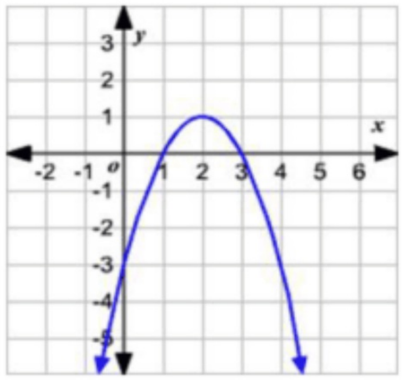
$y \to -\infty$ means "y becomes large in the negative direction"

down.

End Behavior

For example, consider the following graph.





For $y = x$, $y \to \infty$ as $x \to \infty$, and $y \to -\infty$ as $x \to -\infty$

For $y = -x^2$, $y \to -\infty$, as $x \to \infty$, as well as $x \to -\infty$

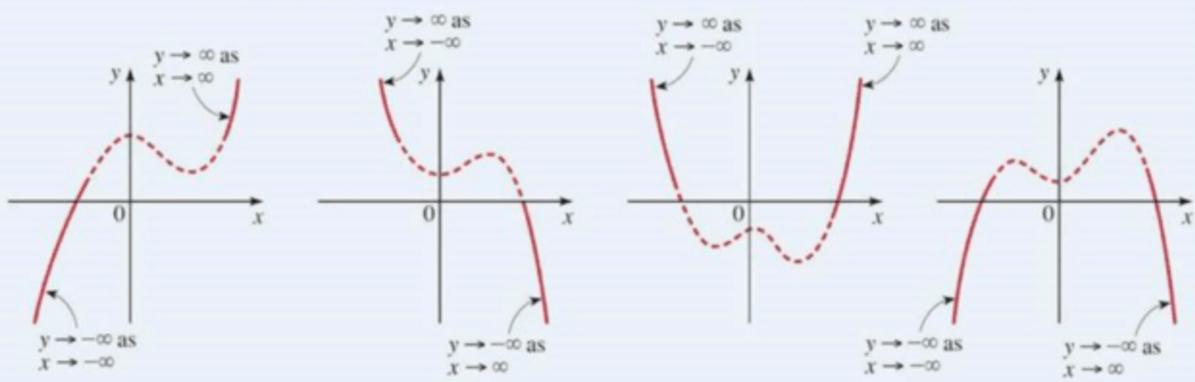
End Behavior

So, we can conclude that polynomials take the following patterns.

END BEHAVIOR OF POLYNOMIALS

The end behavior of the polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ is determined by the degree n and the sign of the leading coefficient a_n, as indicated in the following graphs.

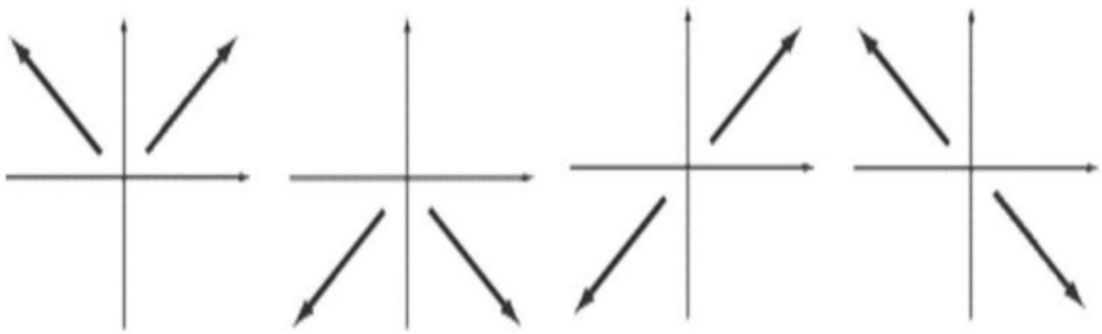
***P* has odd degree** | ***P* has even degree**



Leading coefficient positive | Leading coefficient negative | Leading coefficient positive | Leading coefficient negative

End Behavior

So, we can conclude that polynomials take the following patterns.



n is even	n is even	n is odd	n is odd
$a_n > 0$	$a_n < 0$	$a_n > 0$	$a_n < 0$

Examples

Find the real zeros of the following polynomials and describe their end behaviors.

1. $P(x) = -x^3$

$\sqrt[3]{0} = \sqrt[3]{-x^3}$

$x = 0$

Odd deg.
neg.

$x \to \infty,\ y \to -\infty$
$x \to -\infty,\ y \to \infty$

2. $Q(x) = (x-2)^4$

$\sqrt[4]{(x-2)^4} = \sqrt[4]{0}$

$x - 2 = 0$

$x = 2$

even deg. Positive

$x \to \infty, y \to \infty$

$x \to -\infty, y \to \infty$

3. $R(x) = -2x^5 + 4$

$0 = -2x^5 + 4$

$-4 = -2x^5 \Rightarrow \sqrt[5]{x^5} = \sqrt[5]{2}$

$x = \sqrt[5]{2}$

odd deg neg.

$x \to \infty, y \to -\infty$

$x \to -\infty, y \to \infty$

4. $P(x) = -2x^3 - x^2 + x$

$0 = -x(2x^2 + x - 1)$

$0 = -x(2x-1)(x+1)$

$x = 0, \frac{1}{2}, -1$

odd deg. neg.

5. $P(x) = x^3 + 3x^2 - 4x - 12 = 0$

$x^2(x+3) - 4(x+3) = 0$

$(x^2 - 4)(x+3) = 0$

$x = \pm 2, -3$

odd deg.

pos. $x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty.$

6. $T(x) = x^6 - 2x^3 + 1 = 0$

$(x^3 - 1)(x^3 - 1) = 0$

$x = 1$

even deg.

pos. $x \to \infty, y \to \infty$

$x \to -\infty, y \to \infty$

Homework 10/13

TB pg. 262 #11-35 (e.o.o) Just find the real zeros and describe end behaviors