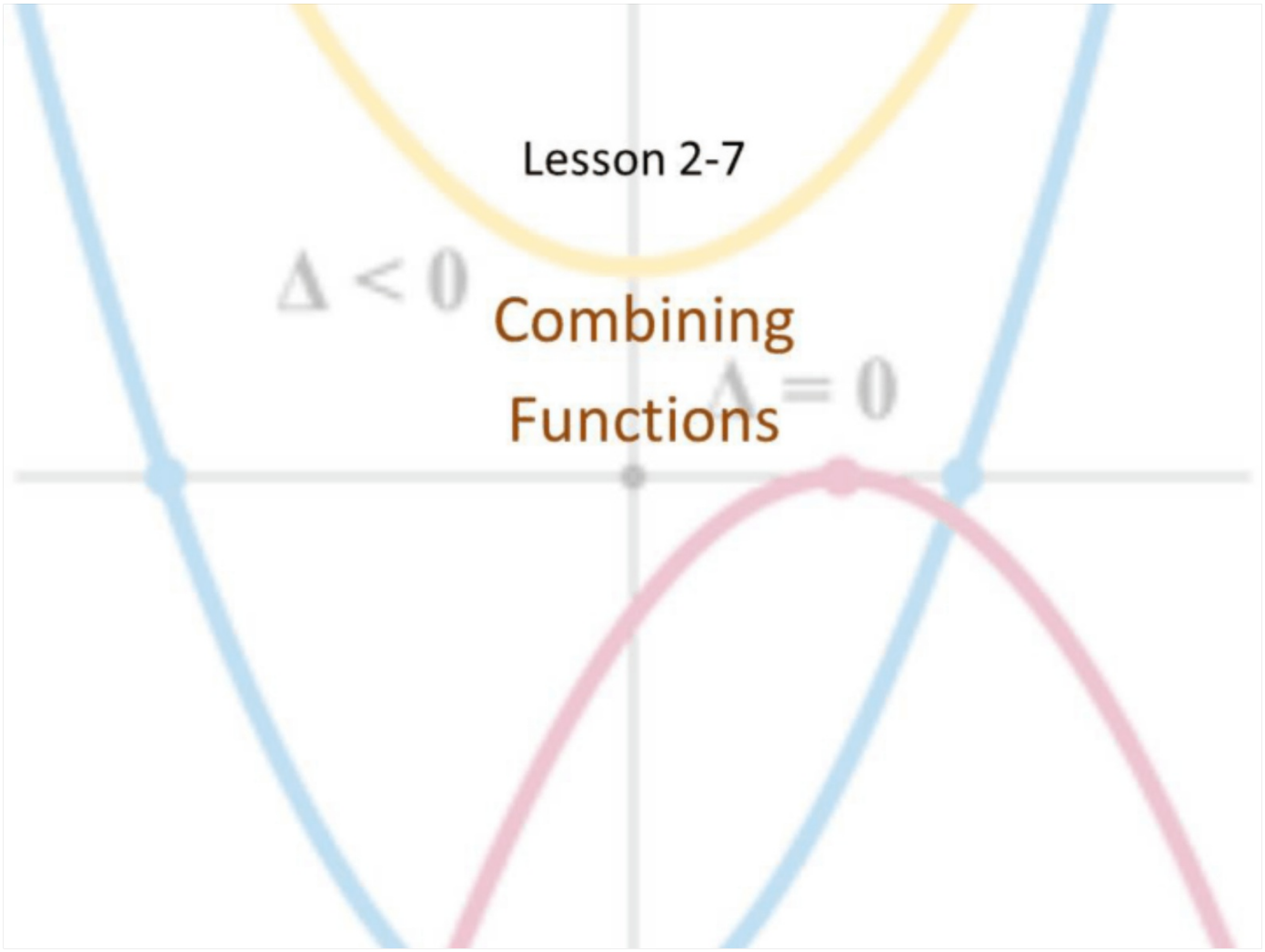
Warm Up 9/26

1. Find the average rate of change of the function between the given values of the variable

a. $f(x) = x^2 - 4; x = 2, x = 3$

b. $g(x) = x^3 - 4x^2; x = 0, x = 10$


Lesson 2-7
$\Delta < 0$
Combining
Functions
$\Delta = 0$

Objective

Students will…

- Be able to add, subtract, multiply, and divide functions.
- Be able to compute the composition of functions.

Adding, Subtracting, Multiplying, and Dividing

There exist sums, differences, products, and quotients within functions. Here are the rules:

Let f and g be functions. Then the functions $f + g, f - g, fg, f/g$ are defined as follows.

$$(f + g)(x) = f(x) + g(x)$$
$$(f - g)(x) = f(x) - g(x)$$
$$(fg)(x) = f(x)g(x)$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Example

Let $f(x) = \frac{1}{x-2}$ and $g(x) = \sqrt{x}$

$g \neq 0$.

a. Find the functions $f + g, f - g, fg, \frac{f}{g}$.

$(f+g)(x) = f(x) + g(x) = \frac{1}{x-2} + \frac{\sqrt{x}}{1} \cdot \frac{(x-2)}{(x-2)} = \frac{1 + \sqrt{x}(x-2)}{x-2}$

$(f-g)(x) = f(x) - g(x) = \frac{1 - \sqrt{x}(x-2)}{x-2}$

$(fg)(x) = f(x)g(x) = \frac{1}{x-2} \cdot \frac{\sqrt{x}}{1} = \frac{\sqrt{x}}{x-2}$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{1}{x-2} \div \frac{\sqrt{x}}{1} = \frac{1}{\sqrt{x}(x-2)}$

Let $f(x) = \frac{1}{x-2}$ and $g(x) = \sqrt{x}$

b. Find $(f+g)(4)$, $(f-g)(4)$, $(fg)(4)$, and $(\frac{f}{g})(4)$

$(f+g)(x) = \frac{1+\sqrt{x}(x-2)}{x-2} \Rightarrow (f+g)(4) = \frac{1+\sqrt{4}(4-2)}{4-2} = \frac{1+4}{2} = \boxed{\frac{5}{2}}$.

$(f+g)(4) = f(4)+g(4) = \frac{1}{4-2} + \sqrt{4} = \frac{1}{2} + \frac{2}{1} = \boxed{\frac{5}{2}}$,

$(f-g)(4) = f(4)-g(4) = \frac{1}{2} - \frac{2}{1} = \boxed{-3/2}$

$(fg)(4) = f(4)g(4) = \frac{1}{2} \cdot \frac{2}{1} = \boxed{1}$

$(\frac{f}{g})(4) = \frac{f(4)}{g(4)} = \frac{1}{2} \div 2 = \boxed{1/4}$

Composition of Functions

With functions, there's a very special way of combining them to get a new function. Consider the following,

Let $f(x) = \sqrt{x}$ and $g(x) = x^2 + 1$

We may define a function h as,

$$h(x) = f(g(x)) = f(x^2 + 1) = \sqrt{x^2 + 1}$$

This is called a composition of functions. The composite function $f \circ g$ (also called a composition of f and g) is defined by

$$(g \circ f)(x) = g(f(x))$$

$$(f \circ g)(x) = f(g(x))$$

$1+2=2+1$
$1\times 2=2\times 1$
$f\circ g\neq g\circ f$.

Example

Let $f(x)=x^2$ and $g(x)=x-3$

a. Find the functions $f\circ g$ and $g\circ f$

b. Find $(f\circ g)(5)$ and $(g\circ f)(7)$

$(f\circ g)(x)=f(g(x))=f(x-3)=(x-3)^2 \Rightarrow f\circ g(5)=(5-3)^2=4$

$(g\circ f)(x)=g(f(x))=g(x^2)=x^2-3 \Rightarrow g\circ f(7)=7^2-3=46$

$(f\circ g)(5)=f(g(5))=f(2)=2^2=4$

$(g\circ f)(7)=g(f(7))=g(49)=46$

Let $f(x) = x^2$ and $g(x) = x - 3$

Find the functions $f \circ f$ and $g \circ g$

$f \circ f = f(f(x)) = f(x^2) = (x^2)^2 = x^4$.

$g \circ g = g(g(x)) = g(x-3) = (x-3)-3 = x-6$

gh

#1. $(g \cdot h)(1)$.

Homework 9/26

Worksheet problems Odd only