

$$\left(\frac{b}{2}\right)^2 = \left(\frac{-2}{2}\right)^2$$

$$= (-1)^2 = 1$$

Warm Up 9/22

1. Complete the square: $f(x) = \frac{-4x^2}{-4} + \frac{8x}{-4} - \frac{2}{-4}$

$$\Rightarrow \frac{f(x)}{-4} = x^2 - 2x + \frac{1}{2} = (x^2 - 2x + 1) + \frac{1}{2} = (x-1)^2 + \frac{1}{2}$$

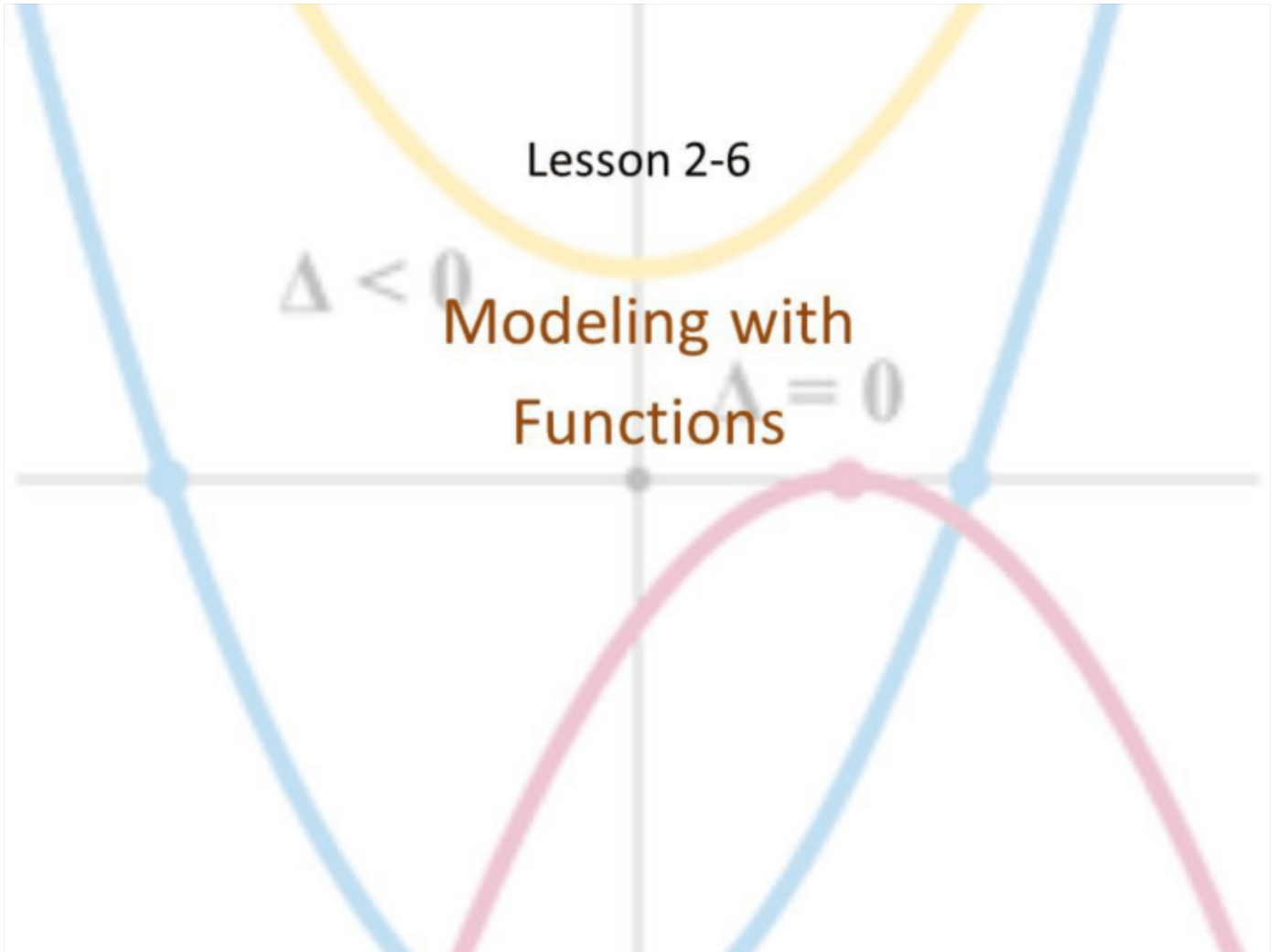
$$\Rightarrow \frac{f(x)}{-4} = \left[(x-1)^2 - \frac{1}{2} \right] \Rightarrow f(x) = -4(x-1)^2 + 2$$

Lesson 2-6

$\Delta < 0$

Modeling with
Functions

$\Delta = 0$



Objective

Students will...

- Model real-life word problems using quadratic functions.
- Be able to solve real-life word problems using functions.

Modeling with Functions

We saw in our previous lesson that quadratic functions can be used to solve real-life related problems, by observing and studying its behavior.

Before, we were given with a function that modeled different situations, although this process is by far the most difficult of all. This section, we learn how to model some of the real-life situations using algebraic and geometric properties.

Lets start with a problem...

A breakfast cereal company manufactures boxes to package their product. For aesthetic reasons, the box must have the following proportions: Its width is 3 times its depth and its height is 5 times the depth.

- a. Find a function that models the volume of the box in terms of depth.

$$V = \text{depth} \times \text{width} \times \text{height}$$

$$\text{let } d = \text{depth} \cdot \text{width} = 3d \cdot \text{height} = 5d$$

$$V(d) = d \times 3d \times 5d = \boxed{15d^3}$$

Guidelines for Modeling with Functions

You may use the following guidelines to aid you if you wish...

1. Express the model (formula) in words- Ex. Area = length x width
2. Choose the variable- Identify all the variables used to express the function. Key is writing it all using one variable instead of multiple.
3. Set up the model- Once you have it written all under one variable, write the function in mathematical language.
4. Use the model- Hard work is virtually done! You may use the function model to solve other applicable problems.

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- a. Find a function that models the volume of the box in terms of depth.

$$V(d) = 15d^3$$

Now use the model! $V(d) = 15d^3$.

b. If the depth is 1.5 in., what is the volume?

$$V(1.5) = 15(1.5)^3 = 15(3.375) = 50.625 \text{ in}^3$$

c. For what depth is the volume 90 in^3 ?

$$90 = 15d^3 \Rightarrow \sqrt[3]{6} = \sqrt[3]{1^3} \Rightarrow d = \sqrt[3]{6} \approx 1.82 \text{ in.}$$

d. For what depth is the volume greater than 60 in^3 ?

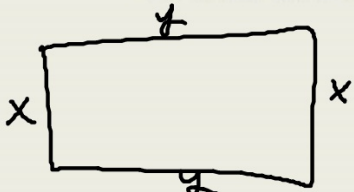
$$\cancel{15d^3} > 60$$

$$\sqrt[3]{6} > \sqrt[3]{4}$$

$$d > 1.58 \text{ in.}$$

A gardener has 140ft of fencing to fence in a rectangular vegetable garden.

a. Find a function that models the area of the garden she can fence.



$$P = 140 \text{ ft.}$$

$$140 = x + x + y + y$$

$$140 = 2x + 2y$$

$$140 = 2(x + y)$$

$$70 = x + y$$

$$y = 70 - x$$

$$A = \text{length} \times \text{width.}$$

$$A = xy$$

$$A(x) = x(70 - x)$$

$$A(x) = 70x - x^2$$

b. For what range of widths is the area greater than or equal to 825ft²?

$$\begin{aligned}
 & \cancel{70x} - x^2 \geq 825 \\
 & -1(-x^2 + 70x - 825) \geq (0) \cdot (-1) \\
 & x^2 - 70x + 825 \leq 0 \\
 & (x-15)(x-55) \leq 0
 \end{aligned}$$

$$\begin{aligned}
 & \cancel{x-15 \leq 0}, \quad \cancel{x-55 \geq 0} \\
 & \cancel{x \leq 15}, \quad \cancel{x \geq 55} \\
 & \cancel{x-15 \geq 0}, \quad \cancel{x-55 \leq 0} \\
 & \boxed{x \geq 15, \quad x \leq 55}
 \end{aligned}$$

c. Can she fence a garden with area 1250ft²?

$$\begin{aligned}
 A(x) &= 70x - x^2 \\
 -x^2 + 70x &= 1250 \\
 -(x^2 + 70x - 1250) &= 0 \cdot (-1) \\
 x^2 - 70x + 1250 &= 0
 \end{aligned}$$

$$\begin{aligned}
 x &= \frac{70 \pm \sqrt{(4900) - 4(1)(1250)}}{2a} \\
 &= \frac{70 \pm \sqrt{4900 - 5000}}{2} =
 \end{aligned}$$

NO

A hockey team plays in an arena with a seating capacity of 15,000. With the ticket price set a \$14, average attendance at recent games has been 9500. A market survey indicates that for each dollar the ticket price is lowered, the average attendance increases by 1000.

- a. Find a function that models the revenue in terms of ticket price.

b. What ticket price is so high that no one attends, and hence no revenue is generated?

c. Find the price that maximizes revenue from ticket sales.

Two ships leave port at the same time. One sails south at 11mi/h and the other sails east at 18mi/h. Find a function that models the distance D between the ships in terms of the time t (in hours) elapsed since their departure.

Find two positive numbers whose sum is 100 and the sum of whose squares is a minimum.

Homework 9/22

TB pg. 210-211 #1, 3, 7, 9, 11, 13,
21, 25