## Lesson 2-4: Transformation of Functions II

## Objective

Students will...

- Be able to apply the properties of reflections in graphing various functions.
- Be able to determine whether a function is even or odd.


## "Parent" Functions

We have seen and studied some of the standard functions and their graphs. For example.

$f(x)=x^{3}$

## Transformation of Functions

Let's go ahead and compare the two functions: $f(x)=x^{2}$ and $g(x)=-x^{2}$

Now let's compare the functions: $f(x)=\sqrt{x}$ and $g(x)=\sqrt{-x}$

## Transformation: Reflection

As observed, the differences between the two functions were either $\qquad$ reflection.
This can be generalized by the following:
Along the y -axis (horizontal)
$y=f(-x)$ reflects the graph of $y=f(x)$ along the $\qquad$ axis ( $\qquad$ reflection).
Along the x -axis (vertical)

$$
y=-f(x) \text { reflects the graph of } y=f(x) \text { along the }
$$

$\qquad$ axis ( $\qquad$ reflection).

Examples
Sketch the following functions by transforming its "parent" function.
a. $f(x)=-|x|$
b. $f(x)=(-x)^{3}$

## Even Functions

Consider the function $f(x)=x^{2}$. We observed that it can be reflected vertically, i.e. along the $x$-axis. What happens when we try to reflect this function horizontally, i.e. along the $y$-axis?
This would mean that the equation would be written in the form of $f(x)=(-x)^{2}=x^{2}$
So, as you can see horizontal reflection really did not change anything. This can easily be seen on its graph.

Any function that has this characteristic is called an $\qquad$ function.


## Odd Functions

Now consider the function $f(x)=x^{3}$. We have already seen it reflected horizontally, i.e. along the $y$-axis. What happens when we reflect this graph vertically, i.e. along the $x$-axis? Look at the graph! Here the graph looks the same whether it is reflected vertically or horizontally.
This can easily be seen algebraically: $(-x)^{3}=-x^{3}$.
Any function that has this characteristic is called an $\qquad$ function.


## Even and Odd Functions

So now we give a formal, generalized definition of even and odd functions:
Let $f$ be a function,

$$
\begin{array}{ll}
f \text { is even if } f(-x)= & , \text { for all } x \text { in the domain of } f \\
f \text { is odd if } f(-x)= & , \text { for all } x \text { in the domain of } f
\end{array}
$$

Ex. Determine whether the following functions are even or odd.
a. $f(x)=x^{5}+x$
b. $g(x)=1-x^{4}$
c. $h(x)=2 x-x^{2}$

