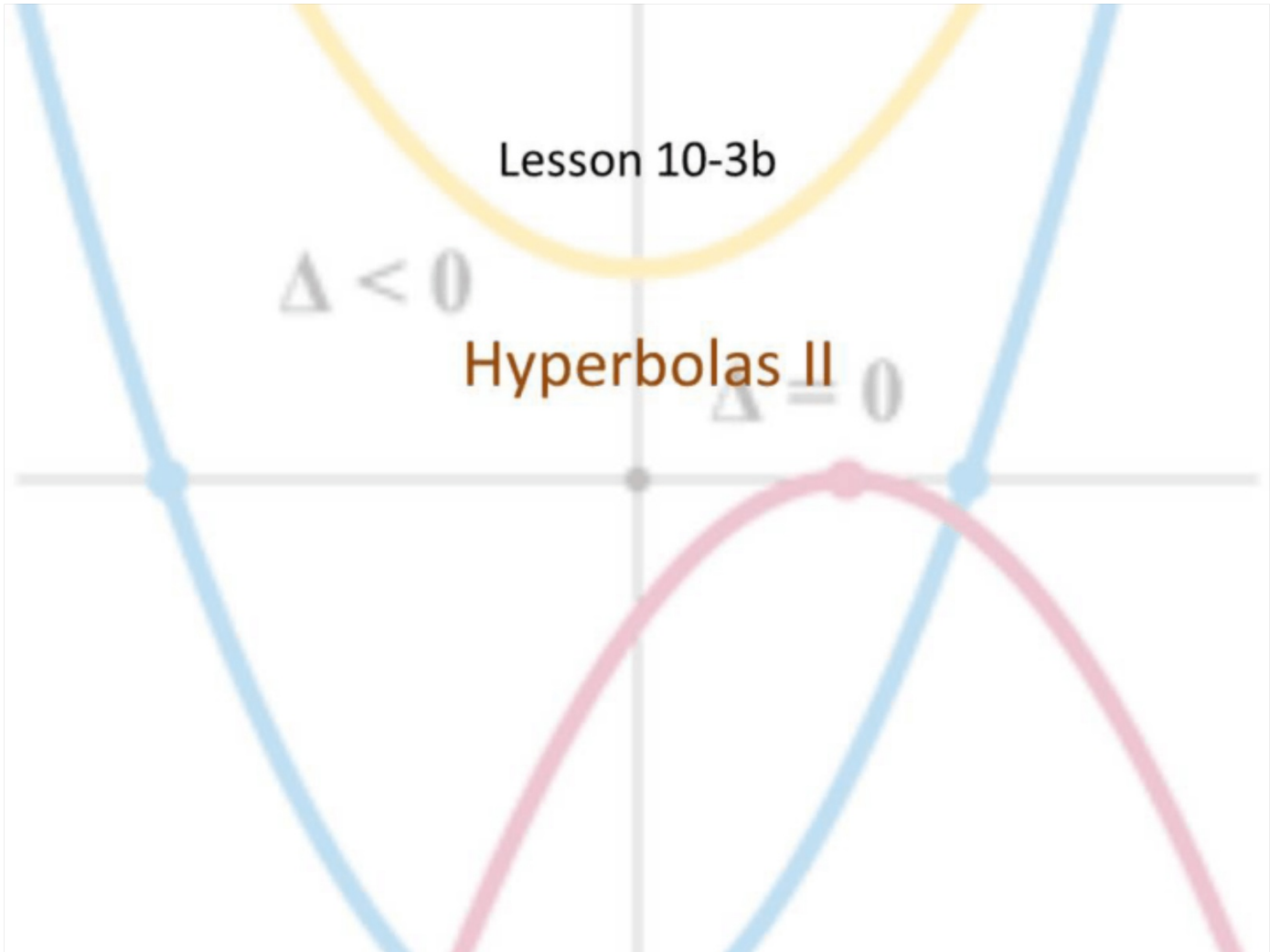


Lesson 10-3b

$\Delta < 0$

Hyperbolas II

$\Delta = 0$



Objective

Students will...

- Be able to derive the equation of standard hyperbolas, given the vertices and foci, and/or the asymptotes.

Equations and Graphs of Hyperbolas

Using the distance formula, we can see that parabolas have the following equations: **for $a > 0$ and $b > 0$ (not $a > b$)**

Horizontal

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

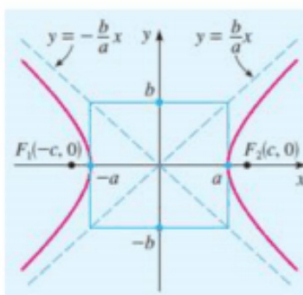
Vertices: $(\pm a, 0)$

Covertices: $(0, \pm b)$

Transverse Axis: Horizontal length $2a$

Asymptotes: $y = \pm \frac{b}{a}x$

Foci: $(\pm c, 0)$, $c^2 = a^2 + b^2$



Vertical

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

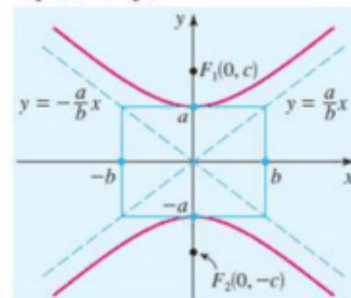
Vertices: $(0, \pm a)$

Covertices: $(\pm b, 0)$

Vertical length $2a$

Asymptotes: $y = \pm \frac{a}{b}x$

Foci: $(0, \pm c)$, $c^2 = a^2 + b^2$

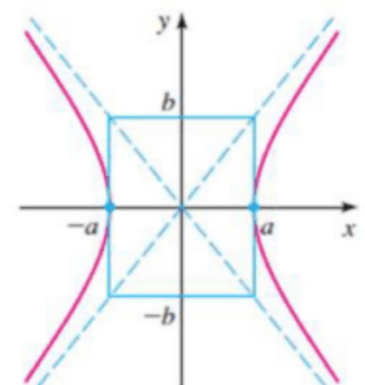
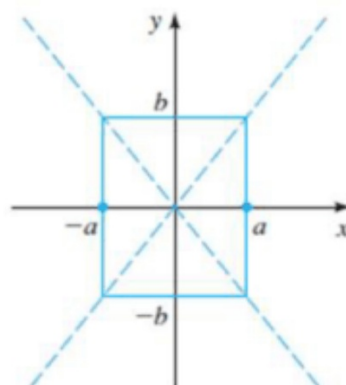
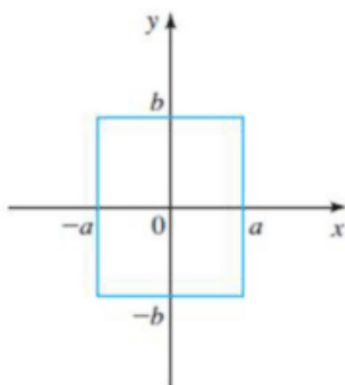


Sketching the Hyperbola

Here is a guidelines you can use to graph the hyperbola.

1. Sketch the **central box**, using the vertices and the covertices.
2. Sketch the **asymptotes**. These are the diagonals of the central box.
3. Plot the foci
4. Sketch the hyperbola.

Ex.



Example

Find the equation of the hyperbola with vertices $(\pm 3, 0)$ and foci $(\pm 4, 0)$. Sketch its graph.

a

$$a=3 \Rightarrow a^2=9$$

$$c=4 \Rightarrow c^2=16$$

$$c^2 = a^2 + b^2$$

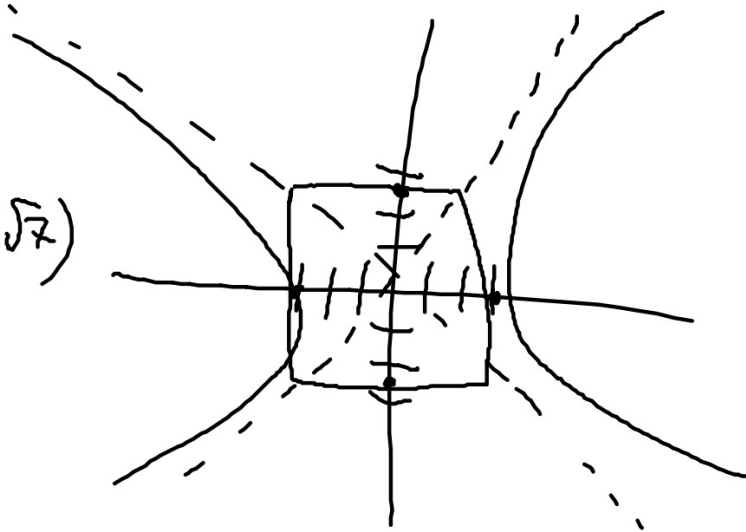
$$16 = 9 + b^2$$

$$7 = b^2$$

(vertices: $(0, \pm\sqrt{7})$)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{horiz.}$$

$$\frac{x^2}{9} - \frac{y^2}{7} = 1$$



Example

Find the equation of the hyperbola with vertices $(\pm 2, 0)$ and foci $(\pm 3, 0)$. Sketch its graph.

$$a^2 = 4$$

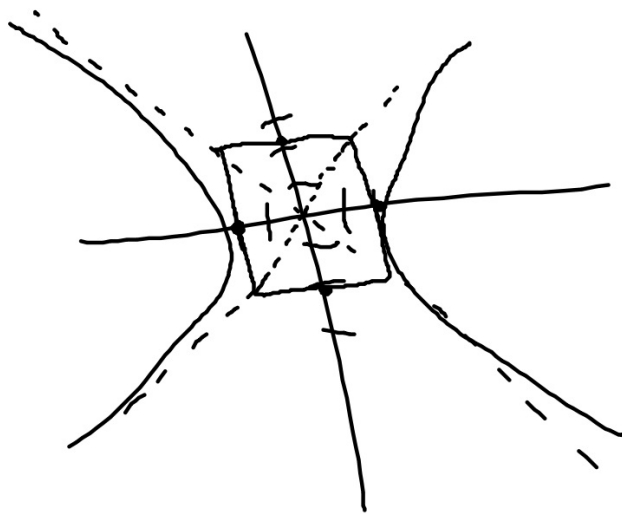
$$c^2 = 9$$

$$b^2 = 5$$

covert: $(0, \pm \sqrt{5})$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{4} - \frac{y^2}{5} = 1$$



Example

Find the equation and the foci of the hyperbola with vertices $(0, \pm 2)$ and asymptotes $y = \pm 2x$. Sketch the graph.

$\swarrow a.$

Vertical

$$a^2 = 4$$

$$b^2 = 1$$

$$c^2 = 5$$

$$c = \pm\sqrt{5}$$

$$\text{Foci: } (0, \pm\sqrt{5})$$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$\frac{y^2}{4} - \frac{x^2}{1} = 1$$

$$\text{asym: } y = \pm \frac{a}{b}x$$

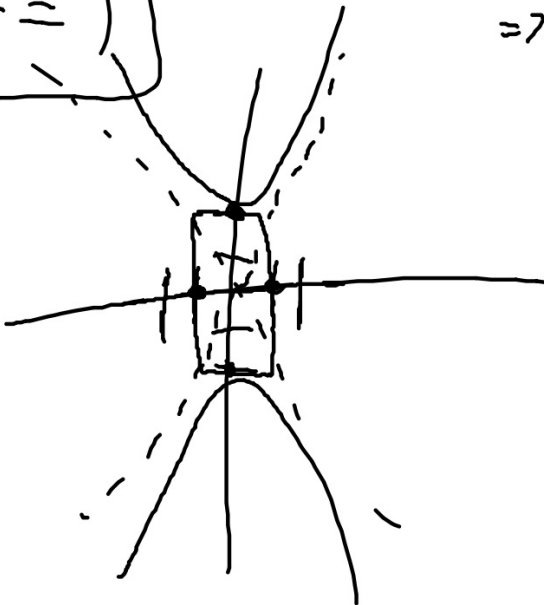
$$\frac{a}{b} = 2.$$

$$\Rightarrow \frac{2}{b} = 2$$

$$2b = 2$$

$$b = 1$$

$$(\pm 1, 0)$$



Example

Find the equation and the foci of the hyperbola with vertices $(0, \pm 4)$ and asymptotes $y = \pm 4x$. Sketch the graph.

$$\frac{y^2}{16} - \frac{x^2}{1} = 1$$

$$y = \pm \frac{a}{b}x$$
$$4 = \frac{4}{1}x$$

Example

Find the equation of the hyperbola with vertices $(0, \pm 6)$, given that it passes through the point $(5, -9)$.

$$a^2 = 36$$

$$\boxed{\frac{y^2}{36} - \frac{x^2}{20} = 1}$$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \Rightarrow \frac{y^2}{36} - \frac{x^2}{b^2} = 1$$
$$\Rightarrow \frac{(-9)^2}{36} - \frac{(5)^2}{b^2} = 1 \Rightarrow \frac{81}{36} - \frac{25}{b^2} = 1$$

$$\Rightarrow \frac{9}{4} - \frac{25}{b^2} = \frac{4}{4} - \frac{9}{4}$$

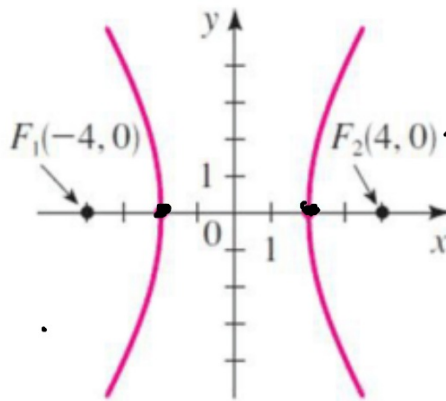
$$\Rightarrow \frac{+25}{b^2} - \frac{+5}{4}$$

$$5b^2 = 100$$
$$\boxed{b^2 = 20}$$

Homework Problem

Find the equation for the hyperbola whose graph is shown.

17.



Foci: $(\pm 4, 0)$
Vert: $(\pm 2, 0)$

$$a = 2 \quad a^2 = 4$$
$$c = 4 \quad c^2 = 16$$
$$b^2 = 12$$

$$\frac{x^2}{4} - \frac{y^2}{12} = 1$$

Homework Problem

Find the equation for the hyperbola with the given conditions.

38. Foci $(0, \pm 1)$, length of the transverse axis 1.

$$c^2 = 1$$

$$\frac{y^2}{\frac{1}{4}} - \frac{x^2}{\frac{3}{4}} = 1$$

$$a = \frac{1}{2}, \quad a^2 = \frac{1}{4}$$
$$b^2 = \frac{3}{4}$$

Homework 5/29

TB pgs. 768-769 #17, 19, 21, 27, 29, 31, 32, 34, 37, 38