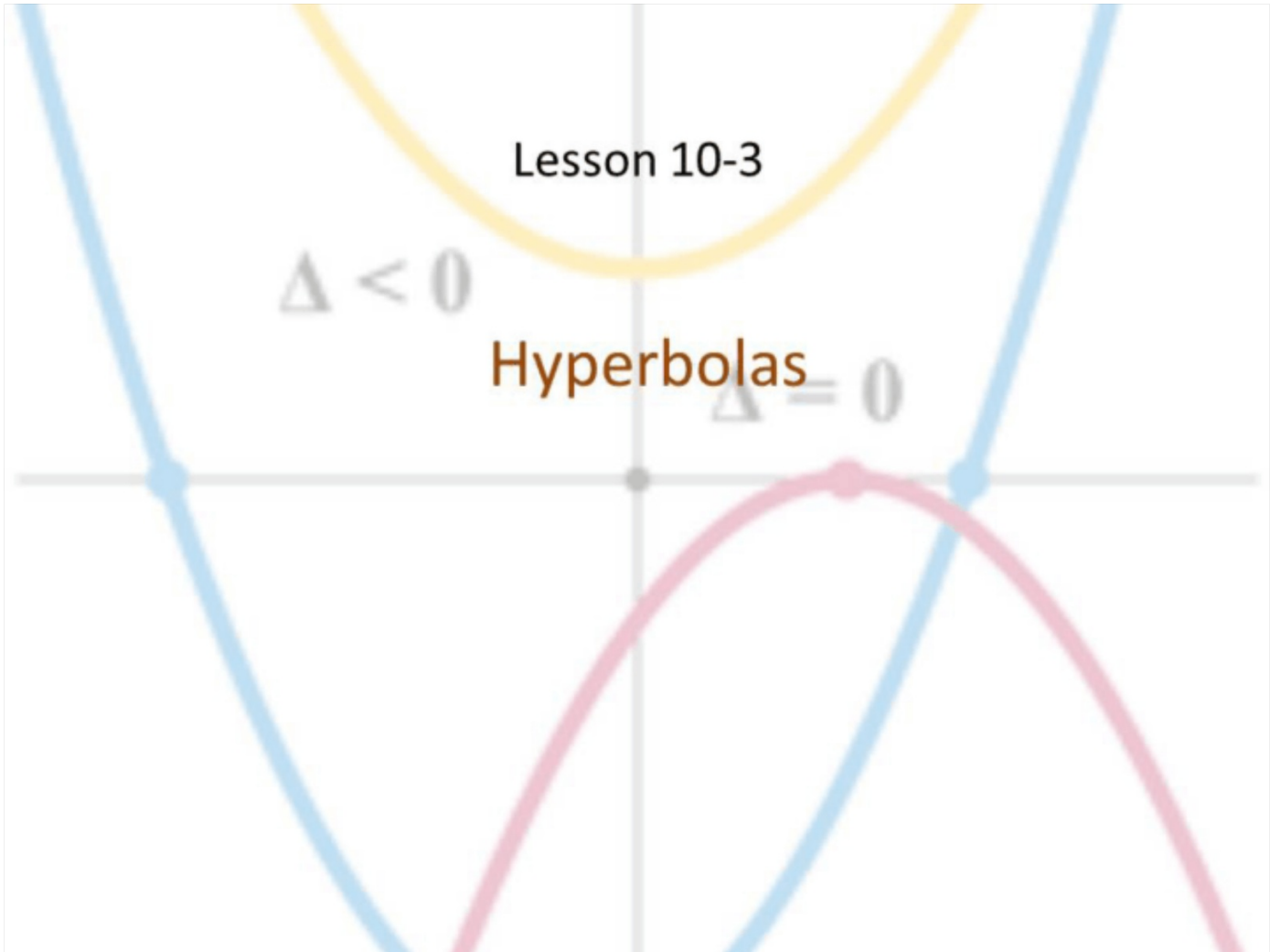


Lesson 10-3

$\Delta < 0$

Hyperbolas

$\Delta = 0$



## Objective

Students will...

- Be able to know the geometric definition of a hyperbola.
- Be able to know and use the standard equation of hyperbolas and sketch their graphs.

## Hyperbolas within a Cone

A hyperbola can be cut out from a cone.



Hyperbola



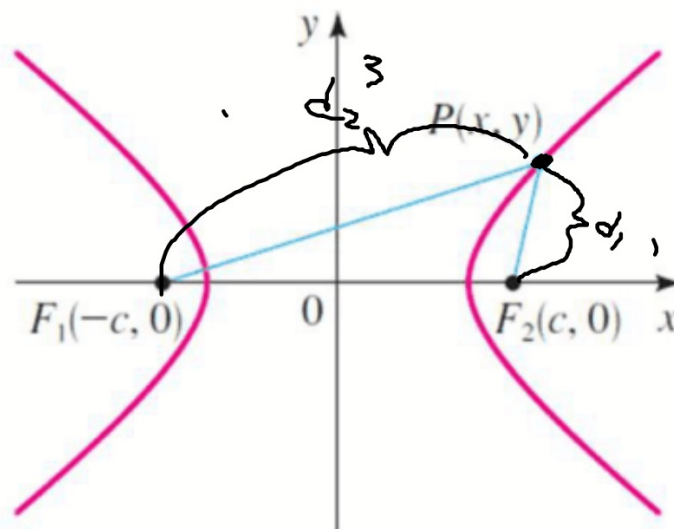
The shape of a cooling tower is a hyperbola.

## Hyperbola

Here, we want to geometrically define what a hyperbola is.

**Geometric Definition of a hyperbola**- Is the set of all points in the plane, the difference of whose distances from two fixed points  $F_1$  and  $F_2$  is a constant. These two fixed points are the foci of the hyperbola.

Ex.



## Equations and Graphs of Hyperbolas

Using the distance formula, we can see that parabolas have the following equations: **for  $a > 0$  and  $b > 0$  (not  $a > b$ )**

### Horizontal

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

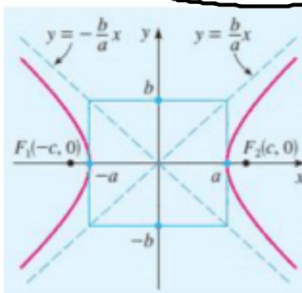
Vertices:  $(\pm a, 0)$

Covertices:  $(0, \pm b)$

Transverse Axis: Horizontal length  $2a$

Asymptotes:  $y = \pm \frac{b}{a}x$

Foci:  $(\pm c, 0)$ ,  $c^2 = a^2 + b^2$



### Vertical

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

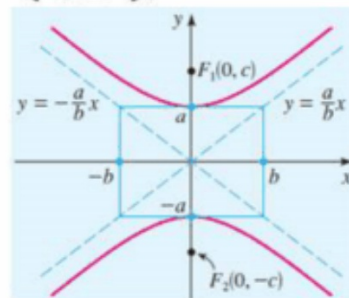
Vertices:  $(0, \pm a)$

Covertices:  $(\pm b, 0)$

Vertical length  $2a$

Asymptotes:  $y = \pm \frac{a}{b}x$

Foci:  $(0, \pm c)$ ,  $c^2 = a^2 + b^2$

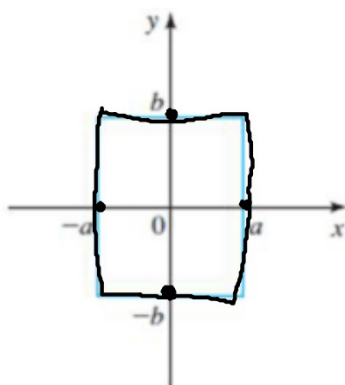


## Sketching the Hyperbola

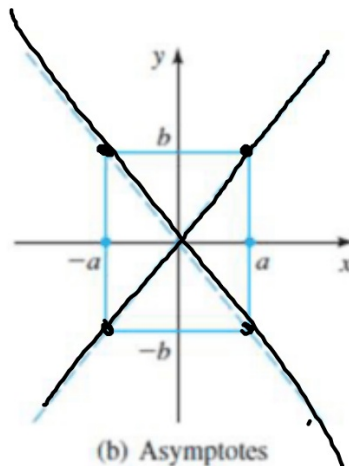
Here is a guidelines you can use to graph the hyperbola.

1. Sketch the **central box**, using the vertices and the covertices.
2. Sketch the **asymptotes**. These are the diagonals of the central box.
3. Plot the foci
4. Sketch the hyperbola.

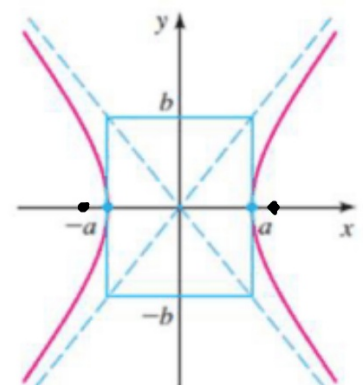
Ex.



(a) Central box



(b) Asymptotes



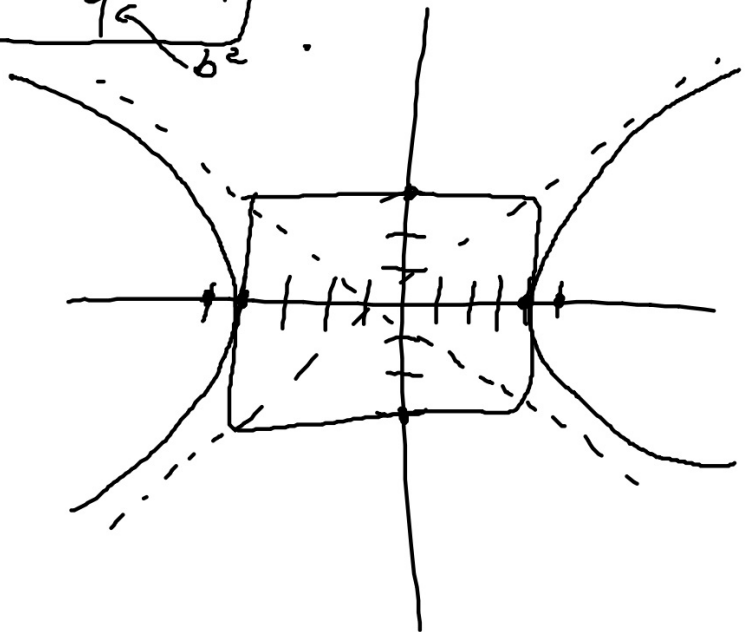
(c) Hyperbola

## Example

Find the vertices, covertices, foci, and the asymptotes. Sketch the graph.

$$\frac{9x^2}{144} - \frac{16y^2}{144} = \frac{144}{144}$$

Vert:  $(\pm a, 0) = (\pm 4, 0)$   $\Rightarrow$   $\frac{x^2}{16} - \frac{y^2}{9} = 1$   
Covert:  $(0, \pm b) = (0, \pm 3)$   
foci:  $(\pm c, 0) = (\pm 5, 0)$   
 $c^2 = a^2 + b^2$   
 $c^2 = 16 + 9 = 25$   
 $c = \pm 5$   
asympt:  $y = \pm \frac{b}{a}x$   
 $y = \pm \frac{3}{4}x$



## Example

Find the vertices, covertices, foci, and the asymptotes. Sketch the graph.

$$\frac{25x^2}{3600} - \frac{144y^2}{3600} = \frac{3600}{3600}$$

$$c^2 = a^2 + b^2$$

$$c^2 = 144 + 25 = 169$$

$$c = \pm 13$$

Vert:  $(\pm 12, 0)$

Covert:  $(0, \pm 5)$

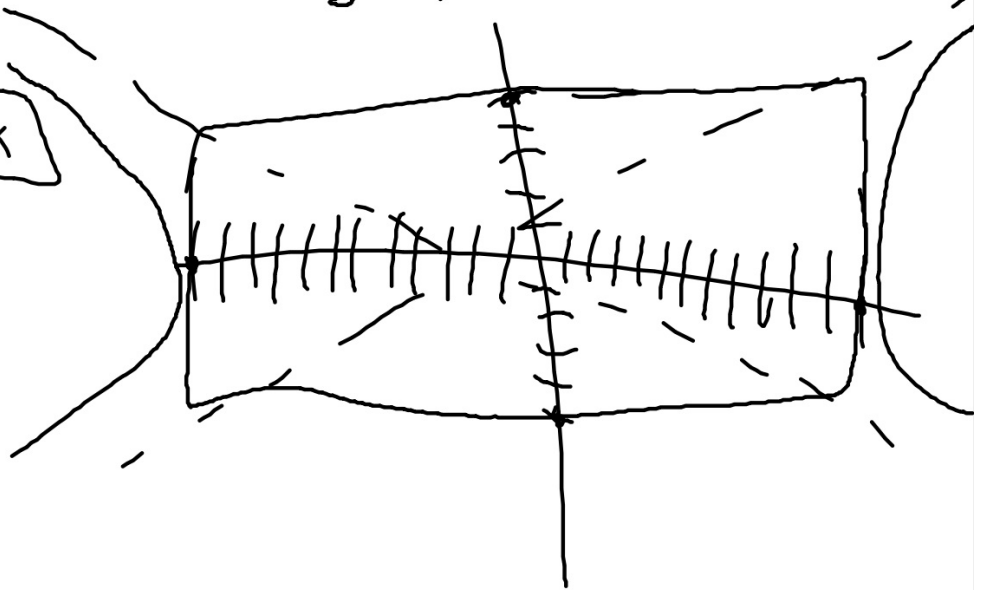
foci:  $(\pm 13, 0)$

asympt:  $y = \pm \frac{5}{12}x$

hori z.

$$\frac{x^2}{144} - \frac{y^2}{25} = 1$$

$a^2$   $b^2$





## Example

Find the vertices, covertices, foci, and the asymptotes. Sketch the graph.

Vert:  $(0, \pm 1)$

Covert:  $(\pm 3, 0)$

foci:  $(0, \pm \sqrt{10})$

asyp:  $y = \pm \frac{1}{3}x$

$$\begin{aligned} x^2 - 9y^2 + 9 &= 0 \\ \frac{x^2}{-9} - \frac{9y^2}{-9} &= \frac{-9}{-9} \end{aligned}$$

$$-\frac{x^2}{9} + \frac{9y^2}{9} = 1$$

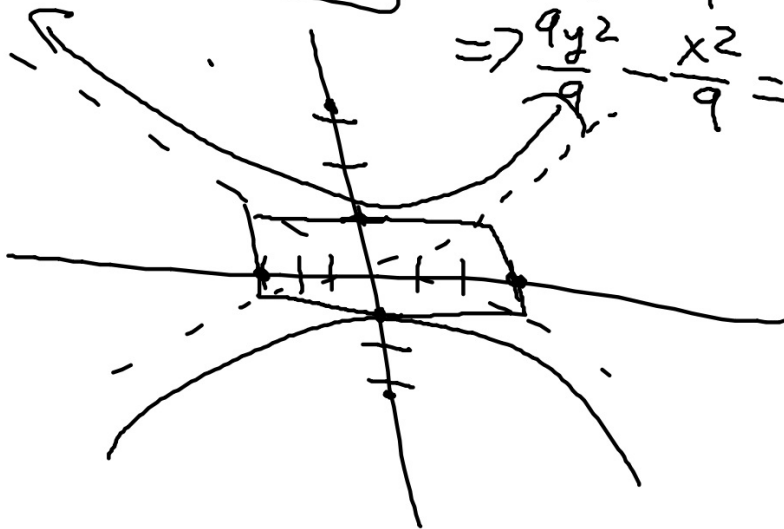
$$\Rightarrow \frac{9y^2}{9} - \frac{x^2}{9} = 1$$

$$\Rightarrow \frac{y^2}{1} - \frac{x^2}{9} = 1$$

$\xrightarrow{a^2} 1$        $\xleftarrow{b^2} 9$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 1 + 9 = 10 \\ c &= \pm \sqrt{10} \approx \pm 3.16 \end{aligned}$$

Vertical



## Example

Find the vertices, covertices, foci, and the asymptotes. Sketch the graph.

$$\frac{y^2}{4} - x^2 = 1$$

## Homework 5/27

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