
Lesson 10-2b
$\Delta < 0$
Ellipses: Eccentricity
$\Delta = 0$

Objective

Students will…

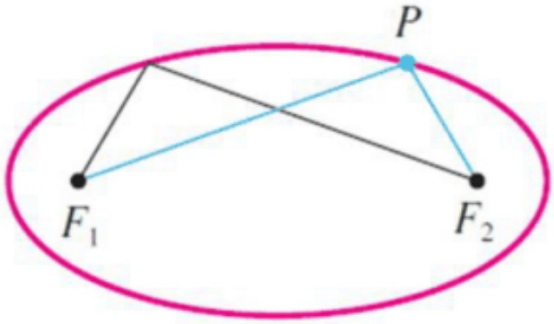
- Be able to know and set up the ratio of eccentricity for any given ellipse.
- Be able to derive equation of an ellipse given the eccentricity and the foci.

Ellipse

Here, we want to geometrically define what an ellipse is.

Geometric Definition of an Ellipse- An ellipse is the set of all points in the plane the sum whose distances from two fixed points F_1 and F_2 is a constant. These two fixed points are **foci** (plural of focus) of the ellipse.

Ex.



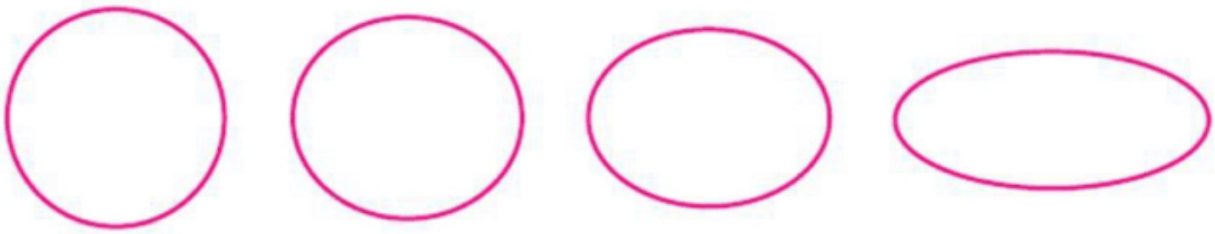
Eccentricity

We know that an ellipse is an oval shape, which can be considered as a **stretched circle**. A numerical value that indicates how close an ellipse is to a circular shape is known as the **eccentricity** of a circle.

Eccentricity- Given a standard ellipse with the equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ or $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, the eccentricity $\boldsymbol{e}$ is the number $e=\frac{c}{a}$, where $c^2=a^2-b^2$ and for every ellipse, $0<e<1$.

Eccentricity of an Ellipse

With that being said, eccentricity is useful in telling us just how close an ellipse looks like a circle. If e is closer to 0, then the ellipse looks more like a circle. If e is closer to 1, then the ellipse looks more like an oval.



$e = 0.1$ $e = 0.5$ $e = 0.68$ $e = 0.86$

Example

Find the eccentricity of the following ellipse.

$$y = \frac{x^2}{25} + \frac{y^2}{9} = 1$$

$a^2 \rightarrow 25$ $\quad b^2 \leftarrow 9$

$c^2 = a^2 - b^2$

$c^2 = 25 - 9$

$c^2 = 16$

$c = 4$

$a = 5$

$e = \frac{c}{a}$

$e = \frac{4}{5} = 0.8$

Example

Find the eccentricity of the following ellipse.

$$y = \frac{x^2}{16} + \frac{y^2}{25} = 1$$

$b^2 \rightarrow$ $\leftarrow a^2$

$a = 5$

$c = 3$

$$e = \frac{c}{a} = \frac{3}{5} = 0.6$$

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Example

(0, ±4)

Find the equation of the ellipse with foci $(0, \pm 8)$ and eccentricity $e = \frac{4}{5} = \frac{c}{a}$

$\frac{4}{5} = \frac{8}{a}$

$4a = 40$

$a = 10$

$\frac{8}{10} = \frac{4}{5}$

$c^2 = 64$

$c = 8$

$a = 10$

$a^2 = 100$

$b^2 = 36$

$$\frac{x^2}{36} + \frac{y^2}{100} = 1$$

$c^2 = a^2 - b^2$

Example

Find the equation of the ellipse with foci $(0, \pm 20)$ and eccentricity $e = \frac{4}{5} = \frac{c}{a}$

$c = 20 \quad c^2 = 400$

~~$a^2 + b^2 = (a+b)^2$~~

$a = 25 \quad a^2 = 625$

~~$5 = 5$~~ $\quad b^2 = 225$

$$\frac{x^2}{225} + \frac{y^2}{625} = 1$$

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TB pg. 759-760 #9-15 (e.o.o) (just find the eccentricity!), 38-40

$\div \frac{3}{2} = 1.5$

11. $\frac{2x^2}{3} + \frac{y^2}{3} = \frac{3}{3}$

$\Rightarrow \frac{2x^2}{3} + \frac{y^2}{3} = 1 \Rightarrow \frac{2}{3}x^2 + \frac{y^2}{3} = 1.$

major: $2(\sqrt{3}) = 2\sqrt{3}$

minor: $2(\sqrt{1.5}) = 2\sqrt{1.5}$

vertices: $(0, \pm\sqrt{3})$

~~Vertex~~

$b^2 \Rightarrow \frac{x^2}{1.5} + \frac{y^2}{3} = 1$ a^2

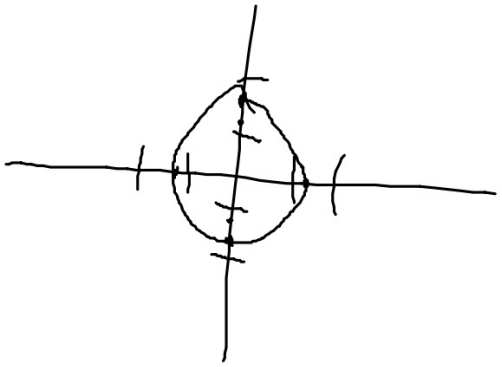
$\Rightarrow \frac{x^2}{\frac{3}{2}} + \frac{y^2}{3} = 1.$

foci: $c^2 = a^2 - b^2$ $(0, \pm\sqrt{1.5})$

$c^2 = 3 - 1.5$

$c^2 = 1.5$

$c = \pm\sqrt{1.5}$



$y^2 \cdot \frac{4}{1} = y^2 \div \frac{1}{4}$

13. $\frac{x^2}{1} + 4y^2 = 1$

$= \frac{x^2}{1} + \frac{y^2}{1/4} = 1$