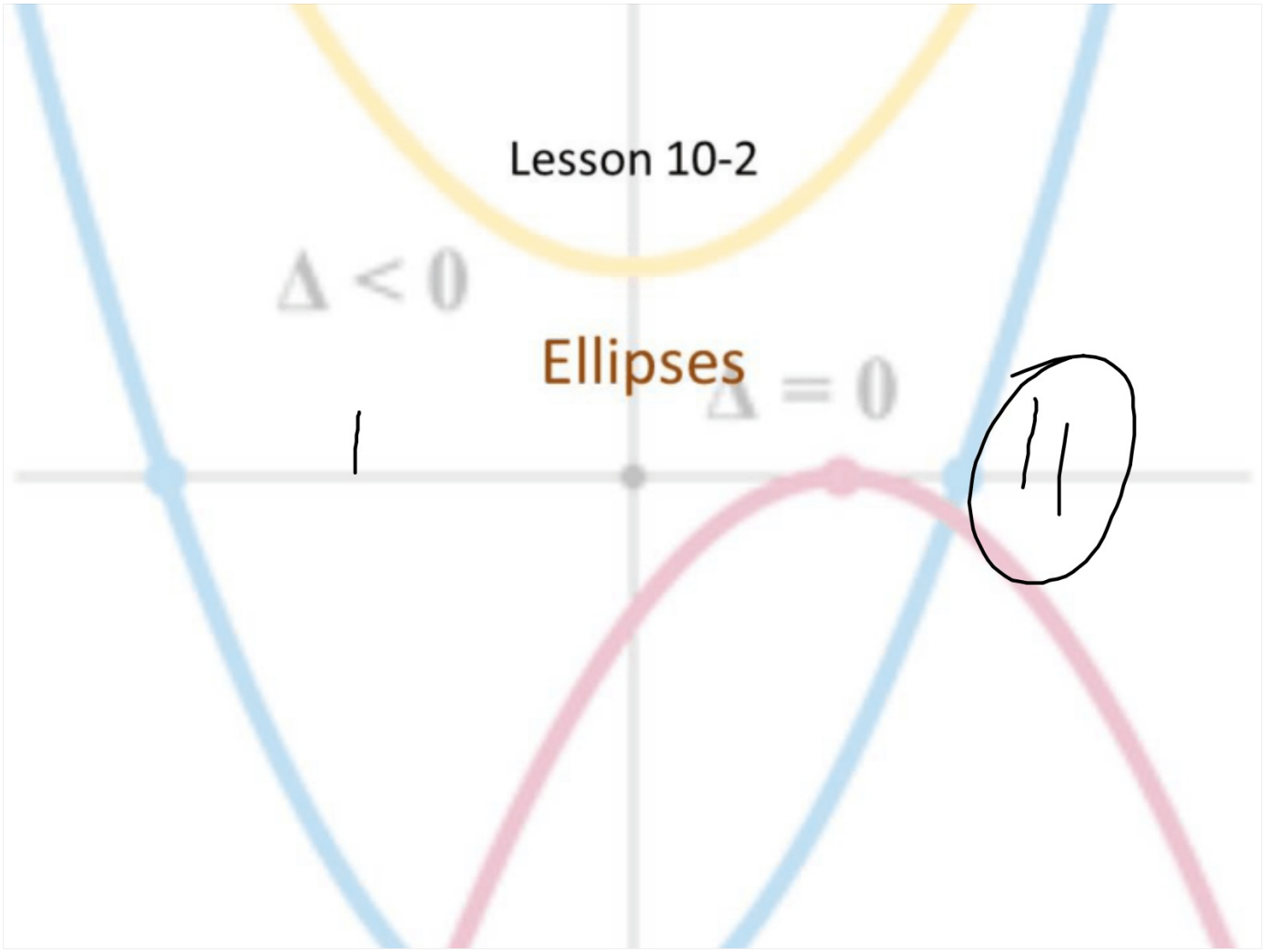

Lesson 10-2
$\Delta < 0$
Ellipses
$\Delta = 0$

Objective

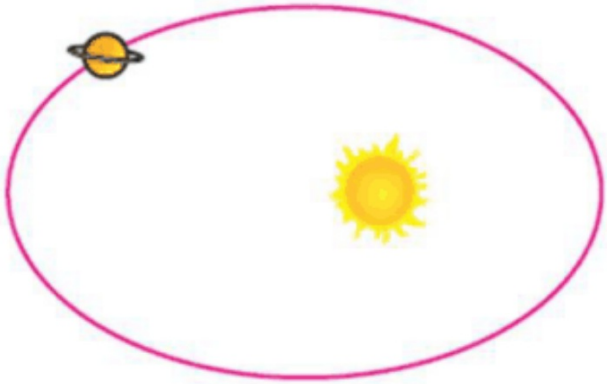
Students will…

- Be able to give a geometric definition of an ellipse.
- Be able to know the standard equation of ellipses.

Ellipse within a Cone

As seen from yesterday's video, a parabola can be cut out from a cone. Parabolas are easily found in the real-world.

Ellipse

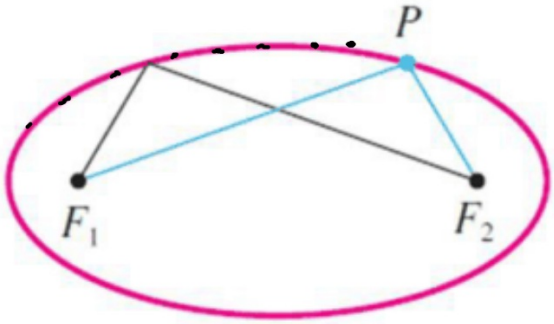


Ellipse

Here, we want to geometrically define what an ellipse is.

Geometric Definition of an Ellipse- An ellipse is the set of all points in the plane the sum whose distances from two fixed points F_1 and F_2 is a constant. These two fixed points are **foci** (plural of focus) of the ellipse.

Ex.



Equations and Graphs of Ellipses

Using the distance formula, we can see that parabolas have the following equations: **for $a > b > 0$,**

Horizontal

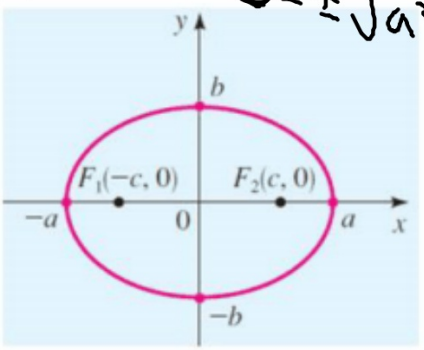
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Vertices: $(\pm a, 0)$ $(a,0),(-a,0)$

Major Axis: Horizontal length $2a$

Minor Axis: Vertical length $2b$

Foci: $(\pm c, 0)$, $c^2 = a^2 - b^2$ $c = \pm\sqrt{a^2-b^2}$



Vertical

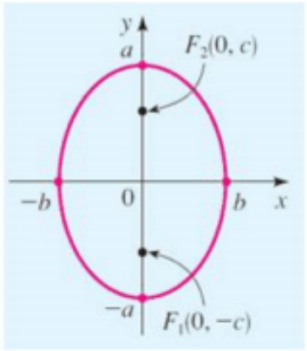
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$(0, \pm a)$

Vertical length $2a$

Horizontal length $2b$

$(0, \pm c)$, $c^2 = a^2 - b^2$



Example

An ellipse has the equation $\frac{x^2}{a^2=9} + \frac{y^2}{4=b^2} = 1$ ← horizontal.

Find the foci, vertices, and the lengths of the major and minor axes, and sketch the graph.

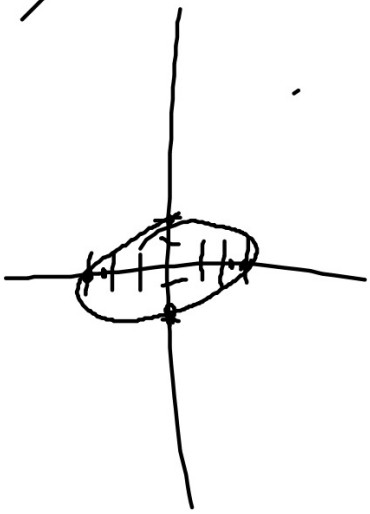
foci: $(\pm c, 0) \Rightarrow c^2 = a^2 - b^2 \Rightarrow c^2 = 9-4 \Rightarrow c = \pm\sqrt{5}$

$= (\pm\sqrt{5}, 0)$ or $(\sqrt{5}, 0), (-\sqrt{5}, 0)$

Vertices: $(\pm a, 0) = (\pm 3, 0)$

maj: horiz. length $2a = 2(3) = 6$

min.: Vertical length $2b = 2(2) = 4$



Example

horiz.

An ellipse has the equation $\frac{x^2}{36} + \frac{y^2}{25} = 1$

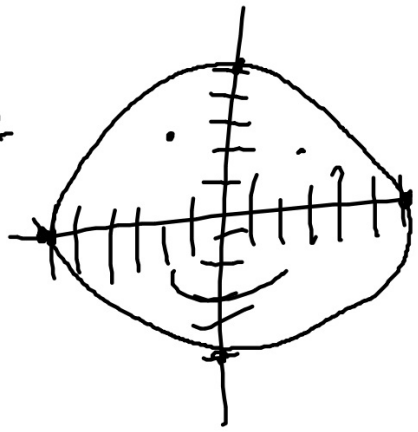
Find the foci, vertices, and the lengths of the major and minor axes, and sketch the graph.

Foci: $(\pm\sqrt{11}, 0)$

Vertices: $(\pm 6, 0)$

maj: horiz. $2(6) = 12$

min: Vert. $2(5) = 10$



Example

Find the foci of the ellipse $16x^2 + 9y^2 = 144$, and sketch its graph.

$\frac{16x^2}{144} + \frac{9y^2}{144} = \frac{144}{144}$ Vertical,

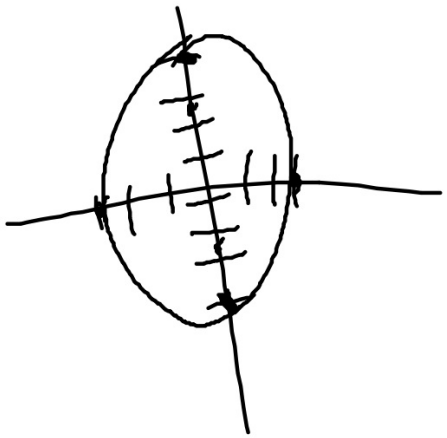
$$\frac{16x^2}{144} + \frac{9y^2}{144} = 1 \Rightarrow \boxed{\frac{x^2}{9} + \frac{y^2}{16} = 1}$$

$c^2 = 16 - 9$

$c^2 = 7 \Rightarrow c = \pm\sqrt{7}$

foci: $(0, \pm\sqrt{7})$

Vertices: $(0, \pm 4)$



Example

horiz. →

The vertices of an ellipse are $(\pm 4, 0)$ and foci are $(\pm 2, 0)$. Find its equation and sketch the graph.

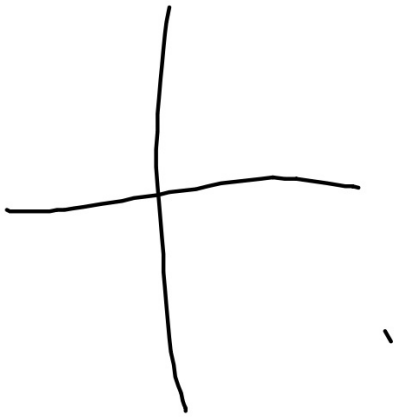
Vertices: $(\pm a, 0)$

$c^2 = a^2 - b^2$.

$4 = 16 - 12$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$= \frac{x^2}{16} + \frac{y^2}{12} = 1$



Homework 5/19

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