**Name:** ________________ **Period:** _____ **Date:** __________

# PreCalculus Practice Midterm

**Answer the following questions.**

1. Define function.

Function is a relation, in which for every input there exists only one output.

2. Describe the end behavior of **any** positive **odd** polynomial function graphs.

$x \to \infty \quad y \to \infty, \quad x \to -\infty \quad y \to -\infty.$

3. Describe the end behavior of **any** negative **odd** polynomial function graphs.

$x \to \infty \quad y \to -\infty, \quad x \to -\infty \quad y \to \infty.$

4. Let $f(x) = 2x^2 + 8x - 1$

$* \ (ab)^2 = a^2b^2.$

a. $f(1)$

$2(1)^2 + 8(1) - 1$

$2 + 8 - 1$

$\boxed{9}$

b. $f(3)$

$2(3)^2 + 8(3) - 1$

$18 + 24 - 1.$ $\boxed{41}$

c. $f(0)$

$\boxed{-1}$

d. $f(ab)$

$2(ab)^2 + 8(ab) - 1$

$\boxed{2a^2b^2 + 8ab - 1}$

5. For the same function $f(x) = 2x^2 + 8x - 1$ from #2,

a. Find its domain.

$(-\infty, \infty)$

b. Complete the square and write it in the vertex form: $f(x) = a(x - h) + k$

add ~~to~~ here — Sub tract here.

$\left(\frac{b}{2}\right)^2 = \left(\frac{4}{2}\right)^2 = 4$

$2(x^2 + 4x - \frac{1}{2}) \Rightarrow 2[(x^2 + 4x + 4) - \frac{1}{2} - 4$

$2[(x+2)^2 - \frac{9}{2}] = \boxed{2(x+2)^2 - 9.}$

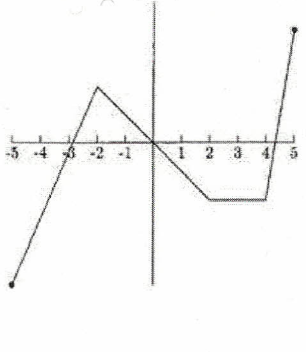
c. Find the vertex and determine whether it's a maximum or a minimum point.

$(-2, -9)$ Minimum

d. Describe the graph's transformation (shift, stretch, compression, etc.) from $f(x) = x^2$

left 2, down 9. Vertical stretch by 2.

6. Use the graph to state the **intervals** in which the function is increasing, decreasing, and neither.



$[-5, -2]$ – increasing

$[4, 5]$ – increasing.

$[-2, 2]$ – decreasing.

$[2, 4]$ – neither.

7. Write the following equation for $y$ in terms of $x$: $3x + 4y = 2$

$4y = 2 - 3x$

$\boxed{y = \frac{1}{2} - \frac{3}{4}x}$

8. Write the following equation for $x$ in terms of $y$: $x - 2y - 3 = 0$

$\boxed{x = 2y + 3}$

$\xrightarrow{}$ Slope

9. For the function $f(x) = 3x - 2$, determine the average rate of change between $x = 2$, and $x = 3$

$f(2) = 3(2) - 2 = 4$

$f(3) = 3(3) - 2 = 7$

$(2, 4)$ $(3, 7)$ $m = \frac{7-4}{3-2} = \frac{3}{1} = \boxed{3}$

10. Determine whether the following functions are one-to-one. If they are, find their inverse function.

a. $f(x) = -2x + 4$

One-to-one

$f(x_1) = f(x_2)$

$-2(x_1) + 4 = -2(x_2) + 4$

$\frac{\cancel{-2}x_1}{\cancel{-2}} \cancel{+4} = \frac{\cancel{-2}x_2}{\cancel{-2}} \cancel{+4}$

$-4 \quad -4$

$x_1 = x_2$ ✓ one-to-one

inv.: $y = -2x + 4$

$x = -2y + 4$

$-4 \quad -4$

$\frac{x-4}{-2} = \frac{\cancel{-2}y}{\cancel{-2}}$

$\boxed{y = \frac{-x-4}{2}}$

b. $f(x) = x^2 - 2x$

$f(x_1) = f(x_2)$

$x_1^2 - 2x_1 = x_2^2 - 2x_2$

even exponent

So not one-to-one

c. $g(x) = \sqrt{x}$

$g(x_1) = g(x_2)$

$(\sqrt{x_1})^2 = (\sqrt{x_2})^2$

$x_1 = x_2$ ✓

$y = \sqrt{x}$

$x = \sqrt{y}$

$\boxed{y = x^2}$

11. Let $f(x) = x - 3$ and $g(x) = 4x^2$. Find $f + g,\ f - g,\ fg,\ \frac{f}{g},\ f \circ g,\ g \circ f$

$f + g = \boxed{x - 3 + 4x^2}$

$f - g = \boxed{x - 3 - 4x^2}$

$fg = (x-3)(4x^2)$

$= \boxed{4x^3 - 12x^2}$

$\frac{f}{g} = \boxed{\frac{x-3}{4x^2}}$

$f \circ g = f(g(x)) = f(4x^2) = \boxed{4x^2 - 3}$

$g \circ f = g(f(x)) = g(x-3) = \boxed{4(x-3)^2 - 3}$

12. The effectiveness of a television commercial depends on how many times a viewer watches it. After some experiments an advertising agency found that if the effectiveness E is measured on a scale 0 to 10, then $E(n) = \frac{2}{3}n - \frac{1}{90}n^2$, where $n$ is the number of times a viewer watches a given commercial. For a commercial to have maximum effectiveness, how many times should a viewer watch it?

Need to find vertex $\rightarrow (n, E(n))$

$n =$ number of times

$E(n) =$ effectiveness

$n = \frac{-b}{2a} = \frac{-2/3}{2(-1/90)} = \frac{-2/3}{-2/90} = \boxed{90/3}$

13. For the function: $T(x) = -x^3 - 3x^2 + 13x + 15$

a. Describe the end behavior.

Negative odd $\Rightarrow$ $x \to \infty, y \to -\infty$ $x \to -\infty, y \to \infty$.

b. List the all of the possible rational zeros it can have.

$$\frac{\text{factors of } 15}{\text{factors of } 1} = \frac{\pm 1, \pm 3, \pm 5, \pm 15}{1} = \boxed{\pm 1, \pm 3, \pm 5, \pm 15}$$

c. Find all of the real zeros and the **state their multiplicity**. (**Hint:** Use synthetic division)

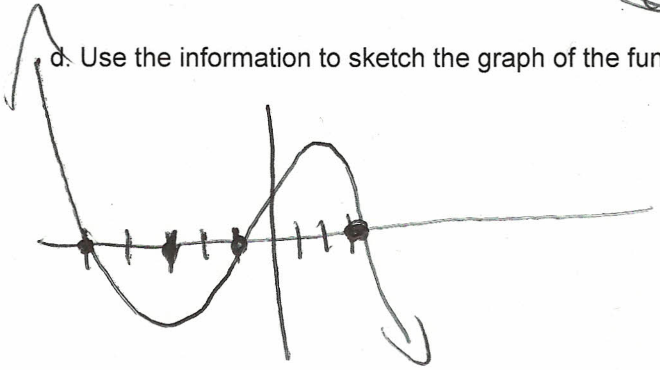
$$\begin{array}{r|rrrr} -1 & -1 & -3 & 13 & 15 \\ & \downarrow & 1 & 2 & -15 \\ \hline & -1 & -2 & 15 & 0 \end{array}$$

$(x+1)(-x^2-2x+15) = (x+1)(x^2+2x-15) = 0$

$(x+1)(x^2+2x-15) = 0 \to (x+1)(x+5)(x-3) = 0$

$$\boxed{\begin{matrix} x = -1, -5, 3 \\ \text{odd odd odd.} \end{matrix}}$$

d. Use the information to sketch the graph of the function.



14. Divide $P(x)$ by divisor $D(x)$ using **long division**.

$P(x) = 9x^2 - x + 5$ $D(x) = 3x^2 - 7x$

$$\begin{array}{r|l} & 3 \\ 3x^2 - 7x & 9x^2 - x + 5 \\ & -\,9x^2 - 21x \quad \downarrow \\ \hline & 20x + 5 \end{array}$$

$$\boxed{9x^2 - x + 5 = 3(3x^2 - 7x) + (20x + 5)}$$

15. For the function: $Z(x) = x^3 - 6x + 4$

a. List the all of the possible rational zeros it can have.

$$\frac{\text{Factors of } 4}{\text{factors of } 1} = \frac{\pm 1, \pm 2, \pm 4}{1} = \boxed{\pm 1, \pm 2, \pm 4}$$

b. Find all of the real zeros. (**Hint:** Use synthetic division. You will be using the quadratic formula)

$$\begin{array}{r|rrrr} +2 & 1 & 0 & -6 & 4 \\ & \downarrow & +2 & 4 & -4 \\ \hline & 1 & +2 & -2 & 0 \end{array}$$

$(x-2)(x^2+2x-2)$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{4-4(1)(-2)}}{2}$$

$$= \frac{-2 \pm \sqrt{4+8}}{2} = \frac{-2 \pm \sqrt{12}}{2} = \frac{-2 \pm 2\sqrt{3}}{2}$$

$$\boxed{2, \pm\sqrt{3}}$$

16. Evaluate the following expressions. Write the result in the form $a + bi$

a. $2i(\frac{1}{2} - i)$

$i - 2i^2$

$= i - 2(-1)$

$= \boxed{2+i}$

b. $(5 - 3i)(1 + i)$

$5 + 5i - 3i - 3i^2$

$5 + 2i - 3(-1)$

$= 5 + 2i + 3$

$= \boxed{8+2i}$

c. $(x - (1 + i))(x - (1 - i))$

$x^2 - x(1-i) - x(1+i) + (1+i)(1-i)$

$= x^2 - x + \cancel{xi} - x - \cancel{xi} + 1 + 1$

$= \boxed{x^2 - 2x + 2}$

d. $\frac{1}{1+i} \cdot \frac{(1-i)}{(1-i)}$

$\frac{1-i}{1+1} = \boxed{\frac{1-i}{2}}$

e. $\frac{2-3i}{1-2i} \cdot \left(\frac{1+2i}{1+2i}\right)$

$= \frac{2 + i - 6i^2}{1 + 4}$

$= \frac{2 + i + 6}{5}$

$= \boxed{\frac{8+i}{5}}$

f. $\frac{-3+5i}{15i} \cdot \frac{-15i}{-15i}$

$= \frac{+45i - 75i^2}{225}$

$= \boxed{\frac{75 + 45i}{225}}$

g. $i^{2024}$

$2 \overline{)2024}$ 1012

$= (i^2)^{1012}$

$= (-1)^{1012}$

$= \boxed{1}$

h. $i^{89}$

$2 \overline{)89}$ 44 R1

$(i^2)^{44} \cdot i$

$= (-1)^{44} \cdot i$

$= \boxed{i}$

17. For the following function: $h(x) = x^3 + x^2 + 81x + 81$

a. **Exactly** how many zeros does it have (real and complex)?

$\boxed{3}$ (the highest exponent)

b. If 1 is the **only** real zero, how many complex zeros does the function have?

$\boxed{2}$ since $1 + 2 = 3$

18. Find a polynomial with integer coefficients that satisfies the given conditions.

a. $P(x)$ with degree 2, and zeros $i$ and $-i$

$(x+i)(x-i) = x^2 \cancel{- xi + xi} - i^2$

$= x^2 - i^2 = \boxed{x^2 + 1}$

c. $T(x)$ with degree 3, and zeros $-3$ and $1 + i$ (Conjugate theorem)

$(x+3)(x - (1+i))(x + (1+i))$

$= (x+3)(x^2 - 2x + 2) \leftarrow \boxed{\text{From \#16 c}}$

$\to x^3 - 2x^2 + 2x + 3x^2 - 6x + 6$

$= \boxed{x^3 + x^2 - 4x + 6}$

~~d. $S(x)$ with degree 1 − 2$i$ and 1, with 1 a zero of multiplicity 2.~~

19. For the rational function $R(x) = \frac{x^2+5x+4}{x-3}$

a. Find the $x$ and the $y$ intercepts.

y-int: $R(0) = \frac{0^2+5(0)+4}{0-3} = \boxed{-\frac{4}{3}}$

x-int: $0 = \frac{x^2+5x+4}{x-3}$

$x^2+5x+4=0$

$(x+4)(x+1)=0$

$\boxed{x=-4,-1}$

b. Find the asymptotes. If no horizontal asymptote exists, then divide to see if slant asymptote exists.

V-Asy: $x-3=0$

$\boxed{x=3}$

H-Asy: Numerator degree is bigger than denom. Hence, No h-asy. exists.

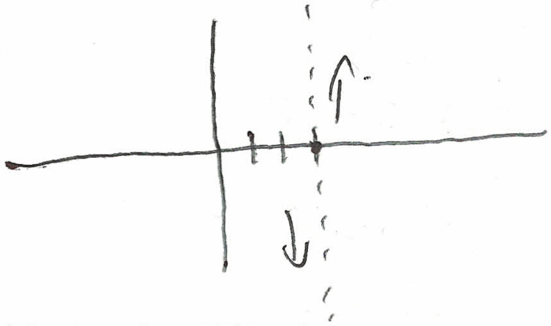
Slant-Asy: $3 \,|\, 1 \;\; 5 \;\; 4$

$\quad\;\; 3 \;\; 24$

$\quad 1 \;\; 8 \;\; \boxed{28}$

$\boxed{y=x+8}$

c. Describe and sketch the behavior of the graph near the vertical asymptotes.



$R(4) = \frac{+}{+} = +$

$R(2) = \frac{+}{-} = -$

20. Evaluate the following, when $f(x) = 2^x$

a. $f(2)$

$2^2 = \boxed{4}$

b. $f(1/2)$

$2^{1/2} = \boxed{\sqrt{2}}$

c. $f(-2)$

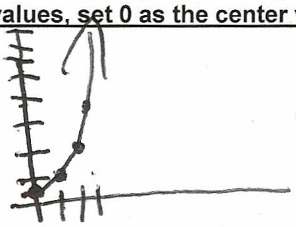
$2^{-2} = \frac{1}{2^2} = \boxed{\frac{1}{4}}$

d. $f(2/3)$

$2^{2/3} = \sqrt[3]{2^2} = \boxed{\sqrt[3]{4}}$

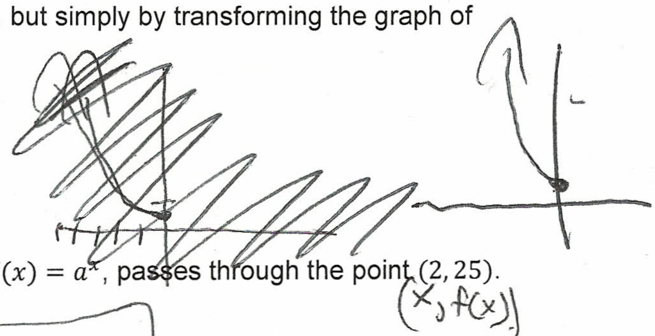
21. For $f(x)$ from #20, graph the function by making a table of values (x-y table).
**<u>(Hint: When choosing the x values, set 0 as the center value)</u>**

| x | y |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |



22. Now, graph $g(x) = \left(\frac{1}{2}\right)^x$, **<u>without</u>** having to make a table, but simply by transforming the graph of $f(x) = 2^x$ from #21.

Vertical reflection..



23. A certain exponential graph of the equation of the form, $f(x) = a^x$, passes through the point $(2, 25)$.
Find the equation of the function (i.e. find $a$)

$(x, f(x))$.

$\sqrt{25} = \sqrt{a^2}$

$\pm 5 = a$

$\boxed{5 = a}$ (only consider positive).

$\boxed{f(x) = 5^x}$.

24. Using a calculator, evaluate the following:

a. $e^2$ $\approx \boxed{7.39}$

b. $e^{-2.4}$ $\approx \boxed{0.09}$

c. $3e^{-5}$ $\approx \boxed{0.02}$

25. Change the following to either logarithmic or exponential form.

a. $\log 100000 = 5$ $\boxed{10^5 = 100000}$

b. $\ln 1 = 0$ $\boxed{e^0 = 1}$

c. $16^{-\frac{1}{4}} = \frac{1}{2}$ $\boxed{\log_{16} \frac{1}{2} = -1/4}$

d. $log_2 \frac{1}{32} = -5$ $\boxed{2^{-5} = \frac{1}{32}}$

26. Use the Laws of Logarithms to either expand or combine the following expressions.

a. $log_2(6x)$

$\boxed{\log_2 6 + \log_2 x.}$

b. $ln\left(\frac{ab}{\sqrt[3]{c}}\right)$

$\ln ab - \ln \sqrt[3]{c}$

$= \ln a + \ln b - \ln c^{1/3}$

$= \boxed{\ln a + \ln b - 1/3 \ln c}$

c. $3\log x + \frac{1}{2}\log(x+1)$

$\log x^3 + \log(x+1)^{1/2}$

$\boxed{\log\left(x^3 (x+1)^{1/2}\right)}$

d. $3\ln s + \frac{1}{2}\ln t - 4\ln(t^2+1)$

$\ln s^3 + \ln t^{1/2} - \ln(t^2+1)^4$

$= \boxed{\ln \frac{s^3 t^{1/2}}{(t^2+1)^4}}$

e. $\log(x^2 y^5)$

$\log x^2 + \log y^5$

$= \boxed{2\log x + 5\log y}$

f. $\log(x) - 3\log(y) + \log(x+y)$

$\log x - \log y^3 + \log(x+y)$

$= \boxed{\log\left(\frac{x}{xy^3+y^4}\right)}$

27. Use the properties and/or Laws of logarithms to evaluate the following: **(No calculator)**

a. $\log 4 + \log 25$

$\log(4 \cdot 25) = \log(100)$

$= \boxed{2}$

b. $log_2 160 - log_2 5$

$\log_2\left(\frac{160}{5}\right)$

$= \log_2(32) = \boxed{5}$

c. $\log(\cancel{\log 10}^{10000})$

$\log(10000)$

$= \boxed{3}$

d. $log_3 100 - log_3 18 - log_3 50$

$\log_3\left(\frac{100}{50} \div 18\right)$

$= \log_3\left(\frac{1}{9}\right) = \boxed{-2}$

~~e. $log_2 6 + log_2 15 + log_2 20$~~

~~$\log_2\left(\frac{3}{5}\right) + \log_2 20 = \boxed{\log_2(12)}$~~

f. $\ln(\cancel{\ln} e^{e^{200}})$

$\ln(e^{200})$

$= \boxed{200}$

28. Solve the following exponential equations.

a. $3^{x+2} = 7$

$\ln 3^{x+2} = \ln 7.$

$(x+2)\ln 3 = \ln 7.$

$x+2 = \frac{\ln 7}{\ln 3}$

$x = \frac{\ln 7}{\ln 3} - 2 = \boxed{-0.23}$

b. $\frac{8e^{2x}}{8} = \frac{24}{8}$

$\ln(e^{2x}) = \ln 3$

$2x = \ln 3$

$x = \frac{\ln 3}{2} = \boxed{0.55}$

~~c. $e^{5-2x}$~~

d. $\log 10^{x^2-5x+7} = \log 1000$

$x^2 - 5x + 7 = \log(1000)$

$x^2 - 5x + 7 = 3$

$x^2 - 5x + 4 = 0.$

$(x-4)(x-1) = 0$

$\boxed{x = 4, 1}$

29. Solve the following logarithmic equations.

a. $\cancel{log_2}(x-3)=3$

$\cancel{2}$ $2$

$x-3=8$

$\boxed{x=11}$

b. $\cancel{\ln} x=3$

$\cancel{e}$ $e$

$x=e^3\approx\boxed{20.1}$

c. $5+4\log(6x)=25$

$-5$ $-5$

$\cancel{4}\log(6x)=20$

$4$

$\cancel{10^{\log}}(6x)=5$

$10$

$6x=10^5$

$\overline{6}$ $\overline{6}$

$x=\frac{10^5}{6}$

$=\boxed{\frac{100000}{6}}$

d. $\cancel{2}\ln(x-2)=6$

$2$ $2$

$\cancel{e^{\ln}}(x-2)=3$

$e$

$x=e^3+2\approx\boxed{22.1}$

e. $\log(x+8)+\log(x-1)=1$

$\log((x+8)(x-1))=1$

$\log(x^2-x+8x-8)=1$

$\cancel{10^{\log}}(x^2+7x-8)=1$

$10$

$x^2+7x-8=10$

$x^2+7x-18=0$

$(x+9)(x-2)=0$

$x=\cancel{-9},\boxed{2}$

f. $\cancel{log_3}(3x^2-8x+11)=3$

$3$

$3x^2-8x+11=27$

$3x^2-8x-16=0$

$(3x+4)(x-4)=0$

$\boxed{x=4,-4/3}$

30. If \$10,000 is invested at an interest rate of 10% per year, find the amount of the investment at the end of 5 years for the following compounding methods. $A(t)=P\left(1+\frac{r}{n}\right)^{nt}$

a. Annually.

$A(5)=10{,}000\left(1+\frac{.1}{1}\right)^{(1\cdot5)}$

$\approx\boxed{\$16105,10}$

b. Quarterly

$A(5)=10000\left(1+\frac{.1}{4}\right)^{20}$

$\approx\boxed{\$16386.16}$

c. Continuously

$A(t)=Pe^{rt}=10000e^{(.1\cdot5)}$

$\approx\boxed{\$16487.21}$

31. A culture contains 1500 bacteria initially and grows at a rate of 99% every minute.

a. Find a function that models the number of bacteria $n(t)$ after $t$ minutes. (growth)

$n(t)=n_0e^{rt}\Rightarrow n(t)=1500e^{(.99\cdot t)}=\boxed{1500e^{.99t}}$

b. Find the number of bacteria after 16 minutes.

$n(16)=1500e^{(.99\cdot16)}\approx\boxed{11,358,365,832}$

c. After how many minutes will the culture contain 15000 bacteria?

$\frac{15000}{1500}=\frac{1500e^{.99t}}{\cancel{1500}}$

$\Rightarrow\ln 10=\cancel{\ln e}^{.99t}$

$\ln 10=.99t$

$t=\frac{\ln 10}{0.99}\approx\boxed{2.3\text{ min.}}$

32. The half-life of a Cesium-137 is 30 years. Suppose that we have a 10-g sample.

a. Find a function that models the mass remaining after $t$ years. (decay)

$m(t)=m_0e^{-rt}$ $r=\frac{\ln 2}{h}=\frac{\ln 2}{30}\approx0.023$

$\boxed{m(t)=10e^{-0.023t}}$

b. How much of the sample will remain after 80 years?

$m(80)=10e^{-0.023(80)}\approx\boxed{1.59\text{ g}}$

c. After how long will only 2-g of the sample remain?

$\frac{2}{10}=\frac{10e^{-0.023t}}{\cancel{10}}$

$\ln\left(\frac{1}{5}\right)=\cancel{\ln e}^{-0.023t}$

$\frac{\ln(1/5)}{-0.023}=t.$

$\boxed{69.98\text{ yrs.}\approx t}$

33. ([✓ T] or F) Only one-to-one functions can have an inverse function.

34. (T or [✓ F]) If a graph stretches vertically, then it also stretches horizontally.

35. ([✓ T] or F) Inverse of the exponential is logarithms

36. ([✓ T] or F) Sine and Cosine curves are periodic.

37. Evaluate the following trigonometric functions. (No calculators!)

a. $\sin\frac{5\pi}{3}$ III $\frac{\pi}{3}=\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ $\boxed{-\frac{\sqrt{3}}{2}}$

b. $\sin\frac{7\pi}{4}$ IV $\frac{\pi}{4}=\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$ $\boxed{-\frac{\sqrt{2}}{2}}$

c. $\cos\frac{5\pi}{6}$ II $\frac{\pi}{6}=\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$ $\boxed{-\frac{\sqrt{3}}{2}}$

d. $\tan\frac{3\pi}{4}$ II $\frac{\pi}{4}=\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$

$\tan=\frac{\sin}{\cos}$

$=\frac{\sqrt{2}/2}{\sqrt{2}/2}=\frac{\sqrt{2}}{2}\cdot\frac{2}{\sqrt{2}}=\boxed{-1}$

e. $\sec\frac{\pi}{3}$ I $\frac{\pi}{3}=\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$

$\sec=\frac{1}{\cos}=\frac{1}{1/2}$

$=\boxed{2}$

38. If $\sin t=\frac{4}{5}$ and $t$ is in quadrant II, find the values of all other trigonometric functions at $t$.

$\cos t=\pm\sqrt{1-\sin^2 t}$

$=\pm\sqrt{1-(4/5)^2}$

$=\pm\sqrt{1-16/25}$ $\rightarrow =\pm\sqrt{9/25}$

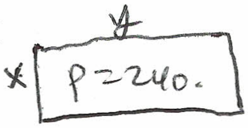
$=3/5$

$\boxed{\cos t=-3/5}$

$\tan t=\frac{\sin t}{\cos t}=\frac{4/5}{-3/5}=\frac{4}{5}\cdot\frac{-5}{3}=\boxed{-4/3}$ $\cot t=\boxed{-3/4}$

$\sec t=\boxed{-5/3}$ $\csc t=\boxed{5/4}$

39. A gardener has 240 feet of fencing in a rectangular vegetable garden. Find the dimensions of the largest area she can fence. What is the maximum area?



$P=2x+2y$

$240=2x+2y$

$240=2(x+y)$

$120=x+y \Rightarrow \boxed{y=120-x}$

$A=xy$ (Find vertex) $x=\frac{-b}{2a}=\frac{-120}{-2}=60$

$A=x(120-x)$ Dimensions: $\boxed{x=60,\ y=60}$

$A=120x-x^2.$ Area: $\boxed{3600}$

40. A hockey team plays in an arena with a seating capacity of 10,500 spectators. With the ticket price set at $10, average attendance at recent games has been 9000. A market survey indicates that for each dollar the ticket price is lowered, the average attendance increases by 1000.

a. What ticket price is so high that no one attends, and hence no revenue is generated?

$R(x)=\text{Attendance}\cdot x.$

Let x= ticket price, Let R(x) represent total revenue. So,

Attendance $=1000(10-x)+9000$

$R(x)=(1000(10-x)+9000)x=(10000-1000x+9000)x=(19000-1000x)x=\boxed{19000x-1000x^2}$

No revenue = 0. $0=19000x-1000x^2$ $\rightarrow x=0,$ $19000-1000x=0$

So, $0=x(19000-1000x)$ $-1000x=-19000$

$\boxed{x=19 \text{ dollars}}$

b. Find the price that **maximizes** revenue from ticket sales.

Find vertex = (ticket price, max revenue) $R(x)=19000x-1000x^2.$

$x=\frac{-b}{2a}=\frac{-19000}{-2000}=19/2.$ $\boxed{\$9.50}$