1) Use: $\frac{d}{dx}(f^{-1}(x)) = \frac{1}{f'(x)}$.

Remember: $f^{-1}(x) = y \iff f(y) = x$.

So, $f^{-1}(\frac{1}{2}) = f(?) = \frac{1}{2}$

By table, $f(\frac{3}{2}) = \frac{1}{2}$.

$\therefore \frac{d}{dx}(f^{-1}(\frac{1}{2})) = \frac{1}{f'(\frac{3}{2})} = \frac{1}{-\frac{1}{2}} = \boxed{-2}$.

2) $\int_0^2 f'(x)\,dx = f(2) - f(0) = \frac{3}{2} - 1 = \boxed{\frac{1}{2}}$.

3) A. Volume increases quicker as the water level rises.

4) $\sum_{n=0}^{\infty} \frac{(2x+1)^{n+1}}{n!} = \frac{2x+1}{0!} + \frac{(2x+1)^2}{1!} + \frac{(2x+1)^3}{2!} + \frac{(2x+1)^4}{3!} + \cdots$

$1 \to$, $1 \to$, $2 \to$, $6 \to$

$\frac{d}{dx}\Sigma = 2 + 4(2x+1) + 3(2x+1)^2 + \frac{4}{3}(2x+1)^3 + \cdots$

$\frac{d^2}{dx^2}\Sigma = 0 + 8 + \cancel{3}3(2x+1) + 8(2x+1)^2 + \cdots$

So, $f''(-\frac{1}{2})$ will make everything w/ $(2x+1) = 0$. Thus, only $\boxed{8}$ remains.

5) Average Value $= \frac{\int_a^b f(x)\,dx}{b-a}$. Since $f(x)$ is cont. & even,

$\int_0^4 f(x)\,dx = \int_{-4}^0 f(x)\,dx$, and $\int_4^6 f(x)\,dx = \int_{-6}^{-4} f(x)\,dx$.

So, Avg. Value $= \frac{2\int_0^4 f(x)\,dx + \int_4^6 f(t)\,dt}{4 - -6} = \frac{2(-5)+2}{10} = \frac{-8}{10} = \boxed{-0.8}$

6) Need to remember: $f'(x)$ tells you the slope, or inc./dec.

$\boxed{f(0) = 1.}$, $f'(0) =$ Negative, $\lim_{h\to 0} \frac{f(h)-1}{h} = \frac{f(0)-1}{0} = \frac{1-1}{0} = \frac{0}{0} \underline{\text{lh}} \lim_{h\to 0} \frac{f'(h)}{1} = f'(0)$,

$\frac{f(1)-f(-1)}{2} = \frac{0-2}{2} = -1$, $\frac{f'(1)-f'(-1)}{2} = \frac{0-0}{2} = 0$.

7) $\lim_{h\to 0}\left(\frac{1}{h}\int_1^{1+h} e^{-t^2}dt\right) = \frac{1}{0}\int_1^{1+0} e^{-t^2}dt = \frac{0}{0}$ = indeterminate.

$= \frac{\int_1^{1+h} e^{-t^2}dt}{h} \overset{\text{lhôpital}}{=} \lim_{h\to 0}\frac{e^{-(1+h)^2} - e^{-(1)^2}}{1} = \frac{e^{-(1+0)^2} - e^{-(1)^2}}{1} = \frac{e^{-1}-e^{-1}}{1} = \frac{0}{1} = \boxed{0}$.

$\frac{d}{dx}\left(\int_a^b f(x)\,dx\right)$

$= \frac{d}{dx}\left(F(b)-F(a)\right)$.

$= f(b) - f(a)$.

9) $\vec{r}''(t) = (t\sin t, \sin t) \Rightarrow r'(t) = (-t\cos t + \sin t + c, -\cos t + c)$.

$\vec{r}'(0) = (0 + \sin(0) + c, -\cos(0) + c) = (c, -1+c) = (0,1)$

So, $\vec{r}'(t) = (-t\cos t + \sin t + 0, -\cos t + 2)$

$\therefore r'(\pi) = (-\pi\cos\pi + \sin\pi, -\cos\pi + 2) = (-\pi(-1), -(-1)+2)$

$= \boxed{(\pi, 3)}$.

10) $u = \sqrt{x}$

$du = \frac{1}{2}x^{-1/2}$

$2\,du = x^{-1/2} = \frac{1}{\sqrt{x}}$

Since we need $\frac{1}{\sqrt{x}}$, $\int \frac{\sqrt{x}}{\sqrt{x}+1}\cdot\left(\frac{\sqrt{x}}{\sqrt{x}}\right) = \int \frac{x}{\sqrt{x}(\sqrt{x}+1)}dx$

So, $\boxed{2\int \frac{u^2}{u+1}\,du}$.

11) $\lim_{x\to\sqrt{3}} \frac{f'(x) - f'(\sqrt{3})}{x-\sqrt{3}} = \frac{f'(\sqrt{3}) - f'(\sqrt{3})}{\sqrt{3}-\sqrt{3}} = \frac{0}{0}$

$\overset{\text{lh}}{=} \lim_{x\to\sqrt{3}} \frac{f''(x) - 0}{1-0} = f''(x) = f''(\sqrt{3})$.

Need to find $f''(x)$.

$\frac{d}{dx}\arctan x = \frac{1}{1+x^2}\cdot x'$

So, $\frac{1}{1+x^2} = (1+x^2)^{-1} = f'(x)$

$\therefore f''(x) = -(1+x^2)^{-2}\cdot 2x$

$\therefore f''(\sqrt{3}) = -(1+(\sqrt{3})^2)^{-2}\cdot 2\sqrt{3} = -(1+3)^{-2}\cdot 2(\sqrt{3}) = \frac{2\sqrt{3}}{-16} = \boxed{-\frac{\sqrt{3}}{8}}$.

12). Since $x=2$ is positive, $|x|-2 = x-2$. So $\frac{x-2}{x-2} = 1$.

Thus, $K=1$.

13) Use Ratio Test: $\frac{(3x+4)^{n+1}}{n+1} \cdot \frac{n}{\cancel{(3x+4)^n}} = \frac{(3x+4)n}{n+1}$.

$\lim_{n\to\infty} \frac{(3x+4)n}{n+1} = (3x+4)\lim_{n\to\infty} \frac{n}{n+1} \overset{lh}{=} \lim_{n\to\infty} \frac{1}{1} = 1$

So, $|3x+4| < 1 \Rightarrow -1 < 3x+4 < 1 \Rightarrow -\frac{5}{3} < x < -1$.

Dist between $-\frac{5}{3}$ and $-1$ $= -1 - -\frac{5}{3} = -\frac{3}{3} + \frac{5}{3} = \frac{2}{3}$.

since radius is half of that, $\boxed{\frac{1}{3}}$.

14) Looking at the graph, $f'(x)$ slope gets closer to zero as $x \to \infty$, $x \to -\infty$. True for every quadrant. So, $\boxed{A}$.

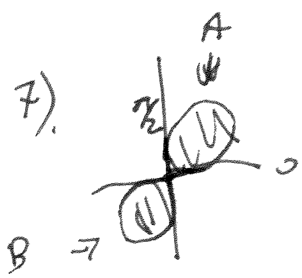
15) VA for $x=0$. For HA, take limit as $x \to \infty$ and $x \to -\infty$. Recall, $\sin(-x) = -\sin x$. So, split into two limits.

$\lim_{x\to\infty} \frac{\sin x}{x} = 0$, and $\lim_{x\to-\infty} \frac{\sin x}{x} = 0$. (By Squeeze Theorem).

Thus, $\boxed{C}$.

16). $f(x) = 4^{3x}$. $f'(x) = \ln(4)(4^{3x}) \cdot 3$.

17).




Area of A = Area of B.

$A = \frac{1}{2}\int_0^{\pi/2} 4\sin 2\theta \, d\theta = \frac{1}{2}\left(4(2)\cos 2\theta \Big|_0^{\pi/2}\right) = 4(\cos\pi - \cos 0)$

$= 4(-1-1)$

$= -8$

18). $\cancel{\sum_{n=1}^{\infty} \frac{n^{1/2}}{n+1}}$ $\cancel{\frac{n^{1/2}}{n} \geq \frac{n^{1/2}}{n+1}}$

By comparison test, $\boxed{\sum_{n=1}^{\infty}\sqrt{\frac{n}{n+1}}}$ Converges: $\sqrt{\frac{n}{n}} \geq \sqrt{\frac{n}{n+1}}$

$\sqrt{\frac{n}{n}} = \sqrt{1} = 1$ ← Convergent.

19) $f(f(x)) = f(e^x) = e^{(e^x)}$, $\frac{d}{dx} e^{e^x} = e^{e^x}\cdot\frac{d}{dx}(e^x) = e^x e^{e^x}$

$= \boxed{e^{x+e^x}}$

20). $Y(\overset{x_0}{1}) = \overset{y_0}{1}$ Need to find $Y$ when $x=2$ ... $h=0.5$, $y' = 1-\frac{x}{y}$.

$Y_1 = Y_0 + h\left(1-\frac{x_0}{y_0}\right) = 1+0.5\left(1-\frac{1}{1}\right) = 1$ $\quad x_1 = x_0 + h = 1+0.5 = 1.5$

$Y_2 = 1 + 0.5\left(1-\frac{1.5}{1}\right) = 1+0.5(1-1.5) = \boxed{0.75}$ $\quad x_2 = x_1+h = 1.5+0.5 = 2$.

21). $\cancel{\frac{d}{dx}\int_x^{2x} \ln t\,dt = \frac{d}{dx}\left(\frac{1}{2x}-\frac{1}{x}\right) = \frac{d}{dx}\left((2x)^{-1}-x^{-1}\right) = -2(2x)^{-2} - -x^{-2} = \frac{-2}{4x^2}+\frac{4}{4x^2} = \frac{2}{4x^2} = \frac{1}{2x}}$

$\frac{d}{dx}\int_x^{2x}\ln t = \ln(2x)-\ln(x) = \ln\left(\frac{2x}{x}\right) = \boxed{\ln(2)}$

22).

23) $\int_1^\infty y\,dx = \lim_{b\to\infty}\int_1^b 10x^{-2}\,dx.$

$= 10\lim_{b\to\infty}\int_1^b x^{-2} = 10\lim_{b\to\infty}\left(-x^{-1}\Big|_1^b\right) = 10\lim_{b\to\infty}\left(-\frac{1}{b} - -\frac{1}{1}\right).$

$= 10\left(\frac{-1}{\infty}+\frac{1}{1}\right) = 10(0+1) = 10.$ Halfway @ $\boxed{5}$.

24) Use the Std. expansion (Need to memorize)

$\frac{x}{1+x^2} = x\,\frac{1}{1+x^2} = x\left(1-x^2+x^4-x^6+\cdots\right)$

$= \boxed{x-x^3+x^5-x^7+\cdots}$

25) $\int_0^2 x\sqrt{4-x^2}\,dx = -\frac{1}{2}\int_0^2 \sqrt{u}\,du = -\frac{1}{2}\left(\frac{2}{3}u^{3/2}\Big|_0^2\right)$

$= -\frac{1}{2}\left(\frac{2}{3}(4-x^2)^{3/2}\Big|_0^2\right)$

$= -\frac{1}{2}\left(0-\frac{8}{3}\right) = \boxed{\frac{4}{3}}$

$u = 4-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

26) Arc length $= \int_a^b \sqrt{1+(y')^2}\,dx.$

$\Rightarrow$ $\frac{4}{x^2} = (y')^2.$

$= \frac{2}{x} = y'$

$\boxed{\ln x^2} = 2\ln(x)$ $= y$

27). $f'(x) = $ slope
Increase most rapidly = largest slope.

$\boxed{C_4}$

28). $x = r\cos\theta$ $\Rightarrow$ $x = \frac{3}{\theta}\cos\theta = 3\theta^{-1}\cos\theta$.
$y = r\sin\theta$ $\quad y = \frac{3}{\theta}\sin\theta = 3\theta^{-1}\sin\theta$

$$\frac{dy}{dx} = \frac{y'}{x'} = \frac{-3\theta^{-2}\sin\theta + 3\theta^{-1}\cos\theta}{-3\theta^{-2}\cos\theta - 3\theta^{-1}\sin\theta}.$$

@ $\theta = \pi/2$ $\Rightarrow$ $$\frac{-3(\frac{\pi}{2})^{-2}\sin(\pi/2) + \cancel{3(\pi/2)^{-1}\cos(\pi/2)}}{\cancel{-3(\pi/2)^{-2}\cos(\pi/2)} - 3(\pi/2)^{-1}\sin(\pi/2)}$$

$$= \frac{-3(2^2)}{\pi^2} \cdot \frac{\pi}{-6} = \boxed{\frac{+2}{\pi}}.$$

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Part B

29). $y$ is the derivative graph.
$\int_x^0$ would be an improper integral

$\int_x^0 = -\int_0^x$. Since $y$ is always inc, $-y$ is always dec

Thus, a, c are out. B is out since there's no plateau of slope in the middle.

The graph should decrease most rapidly in the mid.

D.

30). $f'(x) = \cos(\pi^2) - \cos(0) = -1 - 1 = -2.$

$\cos(4) - \cos(0) =$

31) $y = \sin^2 x$

$\sin y = x.$

$y = \pi/2 \quad x = 1.$

$$\boxed{\pi \int_0^1 \sin^2 y \, dy.}$$