Warm Up 9/26

1. Find the average rate of change of the function between the given values of the variable

a. $f(x) = x^2 - 4; x_1 = 2, x_2 = 3$

$y_1 = 0, y_2 = 5$

$ARC = \boxed{5}$

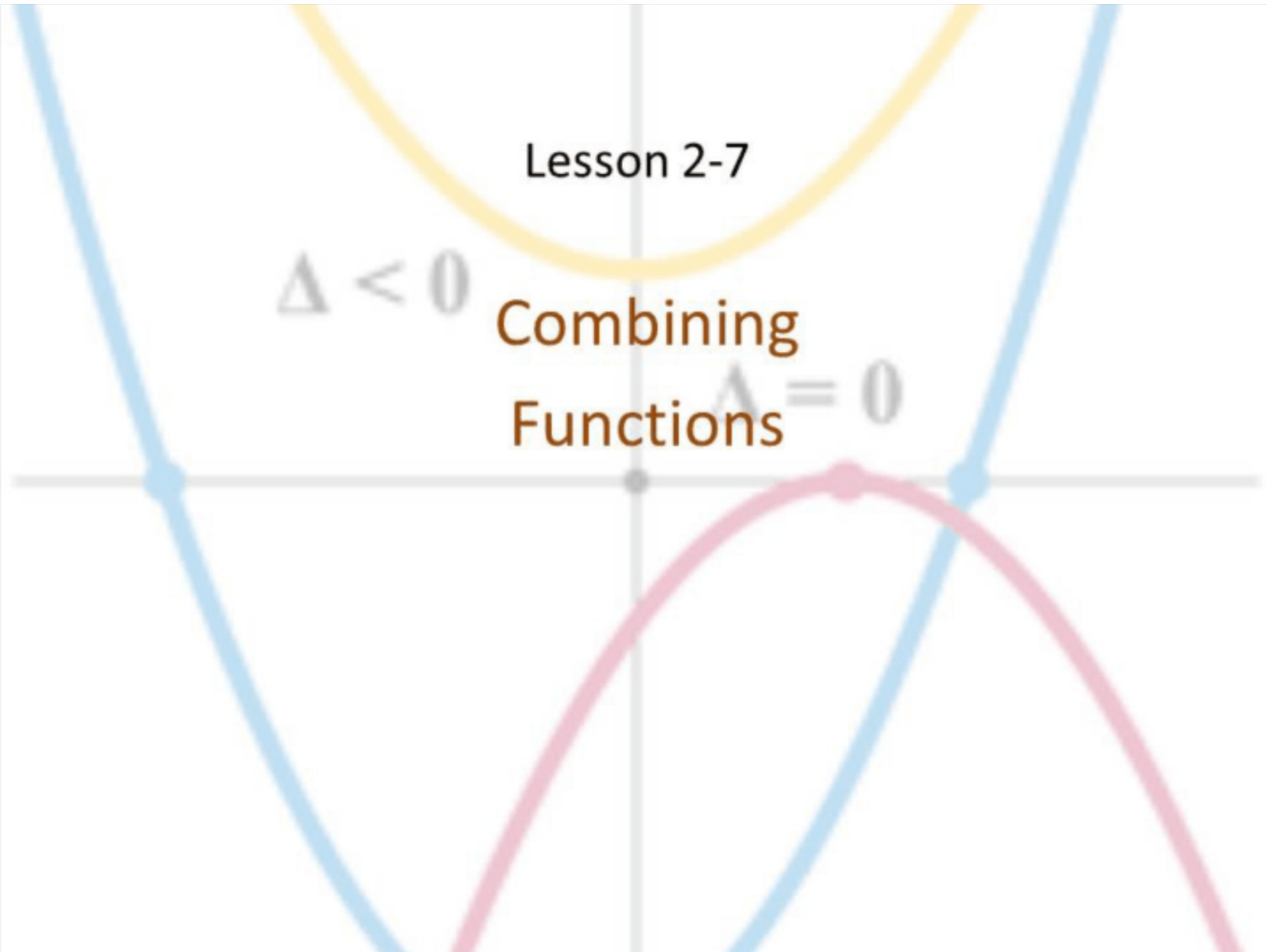
b. $g(x) = x^3 - 4x^2; x_1 = 0, x_2 = 10$

$y_1 = 0\ y_2 = 600$

$\frac{600}{10} = \boxed{60}$

Lesson 2-7

Combining Functions



Objective

Students will…

- Be able to add, subtract, multiply, and divide functions.
- Be able to compute the composition of functions.

Adding, Subtracting, Multiplying, and Dividing

There exist sums, differences, products, and quotients within functions. Here are the rules:

Let f and g be functions. Then the functions $f + g, f - g, fg, f/g$ are defined as follows.

$$(f + g)(x) = f(x) + g(x) = g(x) + f(x) = (g+f)(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x)g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad \text{, where } g(x) \neq 0$$

$f(4) =$

Example

$(f+g)(4) =$

Let $f(x) = \frac{1}{x-2}$ and $g(x) = \sqrt{x}$

a. Find the functions $f+g$, $f-g$, fg, $\frac{f}{g}$

$(f+g)(x) = f(x) + g(x) = \frac{1}{x-2} + \frac{\sqrt{x}}{1} = \boxed{\frac{1+\sqrt{x}(x-2)}{x-2}}$

$(f-g)(x) = f(x) - g(x) = \boxed{\frac{1-\sqrt{x}(x-2)}{x-2}}$

$(fg)(x) = \frac{1}{x-2} \cdot \frac{\sqrt{x}}{1} = \boxed{\frac{\sqrt{x}}{x-2}}$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\frac{1}{x-2}}{\sqrt{x}} = \frac{1}{x-2} \cdot \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}(x-2)}$

$= \boxed{\frac{\sqrt{x}}{x(x-2)}}$

Let $f(x) = \frac{1}{x-2}$ and $g(x) = \sqrt{x}$.

$F(4) = 1/2$ $g(4) = 2$

b. Find $(f+g)(4)$, $(f-g)(4)$, $(fg)(4)$, and $(\frac{f}{g})(4)$ $\frac{5}{2}$ 2.

$(f+g)(4) = f(4) + g(4) = \frac{1}{4-2} + \sqrt{4} = \frac{1}{2} + 2 = 2\frac{1}{2}$

$(f-g)(4) = f(4) - g(4) = \frac{1}{2} - 2 = -1.5 = -3/2$ ..

$(fg)(4) = 1/2 \cdot 2 = 1$

$\left(\frac{f}{g}\right)(4) = \frac{1}{2} \div 2 = 1/4$

Composition of Functions

$f(4) = \sqrt{4}$

With functions, there s a very special way of combining them to get a new function. Consider the following,

Let $f(x) = \sqrt{x}$ and $g(x) = x^2 + 1$

We may define a function h as,

$$= \sqrt{g(x)}.$$

$$f \circ g = h(x) = f(g(x)) = f(x^2 + 1) = \sqrt{x^2 + 1}$$

$$g \circ f = g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 + 1 = x + 1$$

This is called a composition of functions. The composite function $f \circ g$ (also called a composition of f and g) is defined by

$f \circ g \neq g \circ f$

$$(f \circ g)(x) = f(g(x))$$

$$(g \circ f)(x) = g(f(x))$$

$(f \circ g)(5) = f(g(5)) = f(2) = \boxed{4}$

$(g \circ f)(7) = g(f(7)) = g(49) = 49 - 3 = \boxed{46}$

Example

Let $f(x) = x^2$ and $g(x) = x - 3$

a. Find the functions $f \circ g$ and $g \circ f$

b. Find $(f \circ g)(5)$ and $(g \circ f)(7)$

$(f \circ g)(x) = f(g(x)) = f(x-3) = \boxed{(x-3)^2} = \boxed{x^2 - 6x + 9}$

$(g \circ f)(x) = g(f(x)) = g(x^2) = (x^2) - 3 = \boxed{x^2 - 3}$

$(f \circ g)(5) = (5-3)^2 = 2^2 = \boxed{4}$

$(g \circ f)(7) = 7^2 - 3 = \boxed{46}$

Let $f(x) = x^2$ and $g(x) = x - 3$

Find the functions $f \circ f$ and $g \circ g$

$2 + 3 = 5$

$2 + 2$

$(f \circ f)(x) = f(f(x)) = f(x^2) = (x^2)^2 = \boxed{x^4}$

$(g \circ g)(x) = g(g(x)) = g(x-3) = (x-3)-3$

$= \boxed{x-6}$

$f \circ f \circ f = x^6$

$f \circ f \circ f \circ f = x^8$

$g \circ g \circ g \circ g = x - 12$

Homework 9/26

Worksheet problems Odd only