Lesson 3-1

Extrema On An Interval

Objective

Students will…

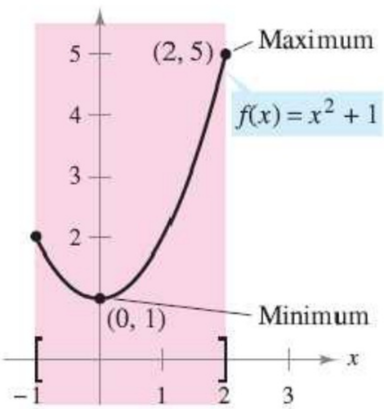
- Be able to understand what an extrema is over an open and closed intervals.
- Be able to distinguish between relative and an absolute extrema.

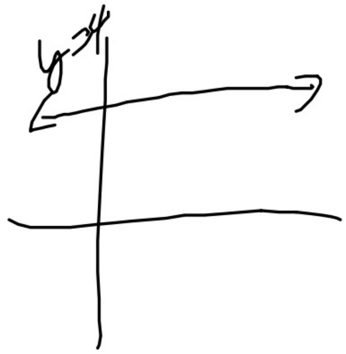
Extrema

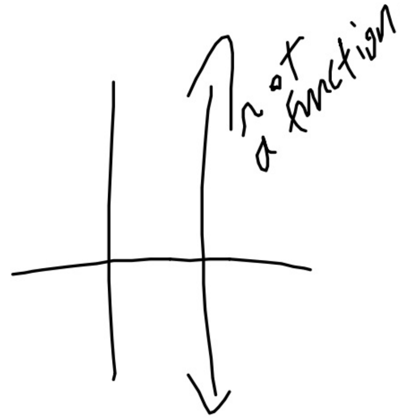
One of the big applications of differentiation is finding the extrema (plural form of the word extremum) of functions. Extrema are the maximum and minimum (extreme) values of a function over an interval.

function

<u>Extreme Value Theorem</u>- If f is continuous on a closed interval $[a, b]$, then f has both a minimum and a maximum on the interval.

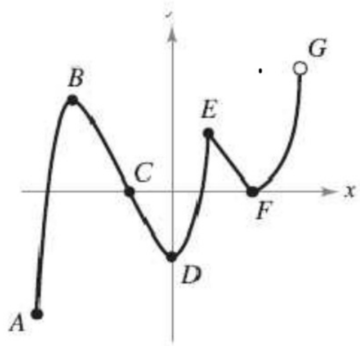






Absolute vs Relative Extrema

A great way to distinguish absolute and relative extrema is to consider whether the interval is open or closed.



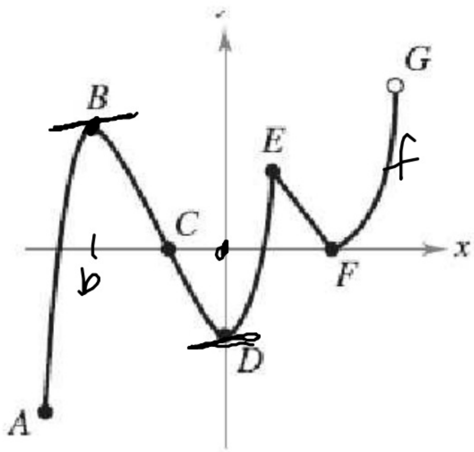
On an open interval (A, G), there are no single extreme values.

On a closed interval $[A, G]$, however, there are single extreme values.

Another good way to identify relative extrema are to think of relative maximum as a hill (or a mountaintop), and relative minimum as a valley.

Application of Differentiation

What connection can we make between derivatives and the relative extrema?



$f'(b) = 0.$

$f'(0) = 0$

$f'(f) = 0$

Application of Differentiation

Overall, we can formulize the steps in finding the extrema.

GUIDELINES FOR FINDING EXTREMA ON A CLOSED INTERVAL

To find the extrema of a continuous function f on a closed interval $[a, b]$, use the following steps.

1. Find the critical numbers of f in (a, b). Values → $f'(x) = 0$ a.k.a. x-int.
2. Evaluate f at each critical number in (a, b).
3. Evaluate f at each endpoint of $[a, b]$.
4. The least of these values is the minimum. The greatest is the maximum.

For open intervals, only do steps 1 and 2.

$xy = 0 \Rightarrow x = 1$
$\text{or } y = 1$

Example

closed

Find the extrema of $f(x) = 3x^4 - 4x^3$ on the interval $[1, 2]$.

$f'(x) = 12x^3 - 12x^2$

CV: $0 = 12x^3 - 12x^2$

$0 = (12x^2)(x - 1)$ (zero product property)

$12x^2 = 0$ or $x - 1 = 0$

$\cancel{x = 0}$ $\qquad x = 1$

$f(1) = 3 - 4 = -1$ abs. min

$f(2) = 48 - 32 = 16$ abs. max

Example

Find the extrema of $f(x) = 2x - 3x^{\frac{2}{3}}$ on the interval $[-1, 3]$.

$f'(x) = 2 - 2x^{-1/3}$

c.v.: $0 = 2 - 2x^{-1/3}$

$\cancel{2}x^{-1/3} = \cancel{2}$

$x^{-1/3} = 1$

$x = 1$

$f(-1) = -2 - 3 = -5$ min.

$f(1) = 2 - 3 = -1$

$f(3) = 6 - 3\sqrt[3]{9} = -0.$ max.

$\frac{11}{3} = 2\frac{5\pi}{3}$ $\frac{5}{3} = 1\frac{2}{3}$ $\frac{11\pi}{\cancel{6}_3} \cdot \frac{\cancel{2}}{1} = \frac{11\pi}{3}$

Example

$2\pi/2 = \pi$ $\frac{7\pi}{\cancel{6}_3} \cdot \frac{\cancel{2}}{1} = \frac{7\pi}{3}$

Find the extrema of $f(x) = 2\sin x - \cos 2x$ on the interval $[0, 2\pi]$.

$f'(x) = 2\cos x + \sin 2x \cdot 2$

CV:

$0 = 2\cos x + 2\sin 2x$

$0 = \cancel{2}(\cos x + \sin 2x)$

$0 = \cos x + 2\sin x\cos x$

$0 = \cos x(1 + 2\sin x)$

$\cos x = 0$ or $1 + 2\sin x = 0$

$x = \pi/2,\ 3\pi/2$

$\sin x = -\frac{1}{2}$

$x = \frac{11\pi}{6},\ 7\pi/6$

$f(0) = 2(0) - 1 = -1$

$f(\frac{\pi}{2}) = 2 - -1 = 3$ max

$f(7\pi/6) = -1 - \frac{1}{2} = -\frac{3}{2}$

$f(3\pi/2) = -2 - -1 = -1$

$f(11\pi/6) = -1 - \frac{1}{2} = -\frac{3}{2}$

min

$f(2\pi) = -1$

Homework 10/23

3.1 Ex #1-2, 3-11 (odd), 13-35 (odd), 37, 39