

Warm Up 9/26

1. Find the average rate of change of the function between the given values of the variable

a. $f(x) = x^2 - 4; x = 2, x = 3$

$$\begin{array}{l} f(2) = 0 \\ f(3) = 5 \end{array} \quad \frac{5-0}{3-2} = \textcircled{5}$$

b. $g(x) = x^3 - 4x^2; x = 0, x = 10$

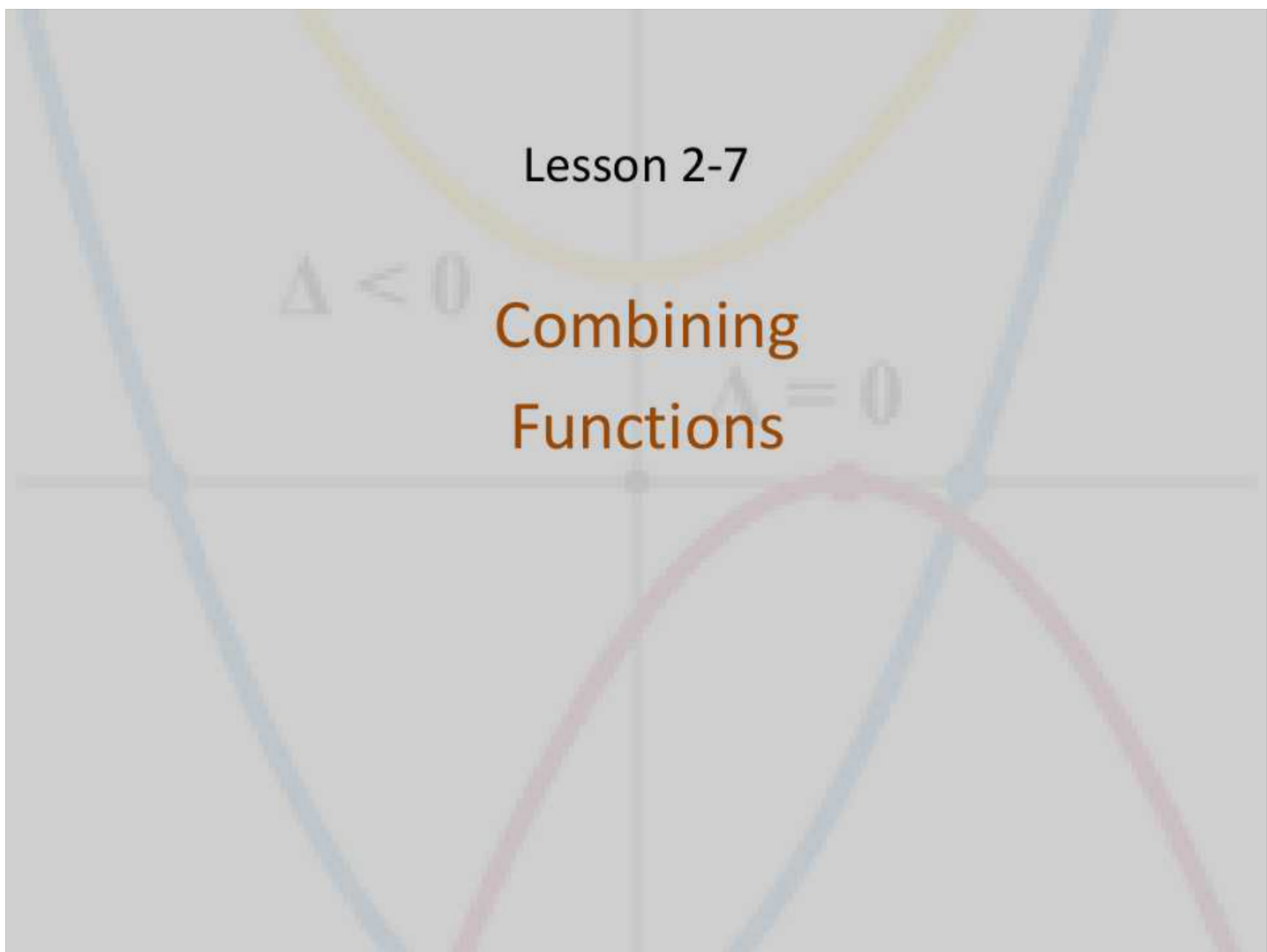
$$\begin{array}{l} g(0) = 0 \\ g(10) = 600 \end{array} \quad \frac{600-0}{10-0} = \textcircled{60}$$

Lesson 2-7

$\Delta < 0$

Combining
Functions

$\Delta = 0$



Objective

Students will...

- Be able to add, subtract, multiply, and divide functions.
- Be able to compute the composition of functions.

Adding, Subtracting, Multiplying, and Dividing

There exist sums, differences, products, and quotients within functions. Here are the rules:

Let f and g be functions. Then the functions $f + g$, $f - g$, fg , f/g are defined as follows.

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x)g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

(where $g(x) \neq 0$.)

Example

Let $f(x) = \frac{1}{x-2}$ and $g(x) = \sqrt{x}$

a. Find the functions $f + g, f - g, fg, f/g$

$$(f+g)(x) = f(x) + g(x) = \frac{1}{x-2} + \frac{\sqrt{x}(x-2)}{1(x-2)} = \frac{1 + x\sqrt{x} - 2\sqrt{x}}{x-2}$$

$$(f-g)(x) = f(x) - g(x) = \frac{1 - x\sqrt{x} + 2\sqrt{x}}{x-2}$$

$$(fg)(x) = f(x)g(x) = \frac{\sqrt{x}}{x-2}$$

$$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{1}{x-2} \cdot \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}(x-2)} = \frac{\sqrt{x}}{x(x-2)}$$

$$f(4) = \frac{1}{2} \quad g(4) = 2$$

Let $f(x) = \frac{1}{x-2}$ and $g(x) = \sqrt{x}$

b. Find $(f+g)(4)$, $(f-g)(4)$, $(fg)(4)$, and $(\frac{f}{g})(4)$

$$(f+g)(4) = \frac{1+4\sqrt{4}-2\sqrt{4}}{4-2} = \frac{1+8-4}{2} = \frac{5}{2} = 2.5$$

$$(f+g)(4) = f(4) + g(4) = \frac{1}{4-2} + \sqrt{4} = \frac{1}{2} + 2 = 2.5$$

$$(f-g)(4) = f(4) - g(4) = \frac{1}{2} - 2 = -1.5$$

$$(fg)(4) = f(4)g(4) = \left(\frac{1}{2}\right)(2) = 1$$

$$\left(\frac{f}{g}\right)(4) = \frac{f(4)}{g(4)} = \frac{\frac{1}{2}}{2} = \frac{1}{4}$$

f(4)

Composition of Functions

← "one inside the other!"

With functions, there is a very special way of combining them to get a new function. Consider the following,

$$\text{Let } f(x) = \sqrt{x} \text{ and } g(x) = x^2 + 1$$

We may define a function h as,

$$h(x) = f(g(x)) = f(x^2 + 1) = \sqrt{x^2 + 1}$$

This is called a composition of functions. The composite function $f \circ g$ (also called a composition of f and g) is defined by

$$(f \circ g)(x) = f(g(x))$$

$$(g \circ f)(x) = g(f(x))$$

$$1+2=2+1$$

Example

$f(7) = 49$ $g(5) = 2$
Let $f(x) = x^2$ and $g(x) = x - 3$

a. Find the functions $f \circ g$ and $g \circ f$

b. Find $(f \circ g)(5)$ and $(g \circ f)(7)$

$$(f \circ g)(x) = f(g(x)) = f(x-3) = \boxed{(x-3)^2}$$

$$(g \circ f)(x) = g(f(x)) = g(x^2) = \boxed{x^2 - 3}$$

$$\begin{aligned} (f \circ g)(5) &= (5-3)^2 = \textcircled{4} \\ &= f(g(5)) = f(2) = 2^2 = \textcircled{4} \end{aligned}$$

$$\begin{aligned} (g \circ f)(7) &= g(f(7)) \\ &= g(49) = \boxed{46} \end{aligned}$$

Let $f(x) = x^2$ and $g(x) = x - 3$

Find the functions $f \circ f$ and $g \circ g$

$$(f \circ f)(x) = f(f(x)) = f(x^2) = (x^2)^2 = x^4$$

$$(g \circ g)(x) = g(g(x)) = g(x-3) = x-3-3 = x-6$$

$$(f \circ f \circ f)(x) = x^8$$

$$(g \circ g \circ g \circ g)(x) = x-12$$

Homework 9/26

Worksheet problems Odd only