$$\left(\frac{b}{2}\right)^2 = \left(\frac{-2}{2}\right)^2$$
$$= (-1)^2 = 1 .$$

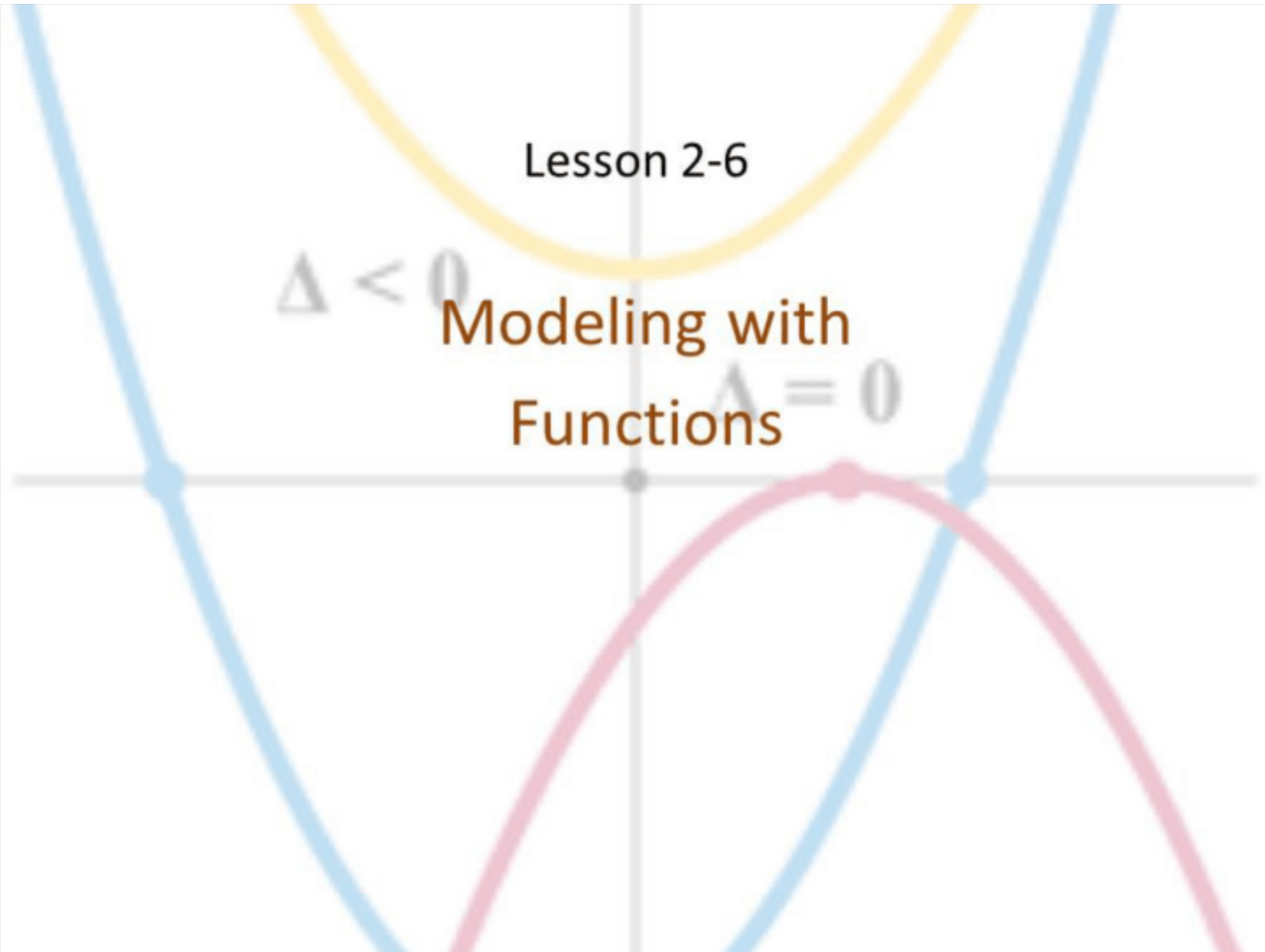
Warm Up 9/22

1. Complete the square: $f(x) = -4x^2 + 8x - 2$

$$\frac{f(x)}{-4} = \frac{-4x^2}{-4} + \frac{8x}{-4} - \frac{2}{-4} \Rightarrow \frac{f(x)}{-4} = x^2 - 2x +$$

$$\Rightarrow \frac{f(x)}{-4} \underset{-1}{+1} = (x^2 - 2x + 1) \cancel{+}\frac{1}{2} = (x-1)^2 + \frac{1}{2} - 1$$

$$\Rightarrow (-4)\frac{f(x)}{-4} = -4\left((x-1)^2 - \frac{1}{2}\right) = \boxed{-4(x-1)^2 + 2} .$$


Lesson 2-6
Modeling with Functions
Δ < 0
Δ = 0

Objective

Students will…

- Model real-life word problems using quadratic functions.
- Be able to solve real-life word problems using functions.

Modeling with Functions

We saw in our previous lesson that quadratic functions can be used to solve real-life related problems, by observing and studying its behavior.

Before, we were given with a function that modeled different situations, although this process is by far the most difficult of all. This section, we learn how to model some of the real-life situations using algebraic and geometric properties.

Lets start with a problem…

A breakfast cereal company manufactures boxes to package their product. For aesthetic reasons, the box must have the following proportions: Its width is 3 times its depth and its height is 5 times the depth.

a. Find a function that models the volume of the box in terms of depth.

Guidelines for Modeling with Functions

You may use the following guidelines to aid you if you wish…

1. <u>Express the model (formula) in words</u>- Ex. Area = length x width

2. <u>Choose the variable</u>- Identify all the variables used to express the function. Key is writing it all using one variable instead of multiple

3. <u>Set up the model</u>- Once you have it written all under one variable, write the function in mathematical language.

4. <u>Use the model</u>- Hard work is virtually done! You may use the function model to solve other applicable problems.

A breakfast cereal company manufactures boxes to package their product. For aesthetic reasons, the box must have the following proportions: Width is 3 times its depth and its height is 5 times the depth.

a. Find a function that models the volume of the box in terms of depth.

V = depth x width x

$$V(d) = d \times 3d \times 5d$$

$$\boxed{V(d) = 15d^3}$$

$V(d) = 15d^3$

Now use the model!

b. If the depth is 1.5 in., what is the volume?

$V(1.5) = 15(1.5)^3 = \boxed{50.625\ in^3}$

c. For what depth is the volume $90in^3$?

$\frac{90}{15} = \frac{\cancel{15}d^3}{\cancel{15}} \Rightarrow \sqrt[3]{d^3} = \sqrt[3]{6} \Rightarrow \boxed{d \approx 1.8\ in.}$

d. For what depth is the volume greater than $60in^3$?

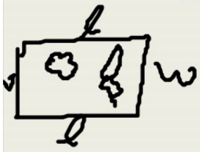
$\frac{60}{15} < \frac{\cancel{15}d^3}{\cancel{15}} \Rightarrow \sqrt[3]{d^3} > \sqrt[3]{4}$

$\Rightarrow \boxed{d > 1.6}$

$P = 140$.

A gardener has 140ft of fencing to fence in a rectangular vegetable garden.

a. Find a function that models the area of the garden she can fence.



Area = length × width.

$$A(w) = (-w + 70)w$$

$$\boxed{A(w) = -w^2 + 70w}$$

$P = 140$

$140 = w + w + l + l$

$140 = 2w + 2l$

$-2w \quad -2w$

$\dfrac{-2w + 140}{2} = \dfrac{2l}{2}$

$l = -w + 70$

$-55 \quad 825 \quad -15$
-70

b. For what range of widths is the area greater than or equal to 825ft^2 ?

$A(w) = -w^2 + 70w$

$825 \le -w^2 + 70w$
$-825 \qquad -825$

$(-1)\, 0 \le -(w^2 + 70w - 825)$

$0 \ge w^2 - 70w + 825$

$0 \ge (w-55)(w-15)$

$\pm \qquad \mp$

~~① $w-55 \ge 0$ and $w-15 \le 0$~~
~~$w \ge 55$ and $w \le 15$.~~

② $w-55 \le 0$ and $w-15 \ge 0$
$w \le 55$ and $w \ge 15$

$\boxed{15 \le w \le 55}$

c. Can she fence a garden with area 1250ft^2 ?

$1250 = -w^2 + 70w$ No
$-1250 \qquad -1250$

$(-1)\, 0 = -(w^2 + 70w - 1250)$

$0 = w^2 - 70w + 1250$

discriminant $b^2 - 4ac$

$(-70)^2 - 4(1)(1250)$

$4900 - 5000$
$= -100$

No sol.

7 A hockey team plays in an arena with a seating capacity of 15,000. With the ticket price set at $14, average attendance at recent games has been 9500. A market survey indicates that for each dollar the ticket price is lowered, the average attendance increases by 1000.

a. Find a function that models the revenue in terms of ticket price.

Let ticket price $= x$

Revenue = ticket $ • Attend

$R = x \cdot \left[(14-x)(1000) + 9500\right]$.

$$\boxed{R(x) = 23500x - 1000x^2}$$

$x\,(14000 - 1000x + 9500)$

$x\,(23500 - 1000x)$

$14 \to 9500

(-1) $13 → 9500+10

(-2) $12 → 9500+2(

(-3) $11 → 9500 +3(

b. What ticket price is so high that no one attends, and hence no revenue is generated?

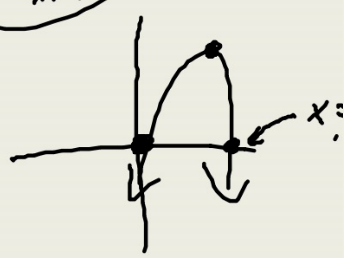
X-int.

$$0 = 23500x - 1000x^2$$

$$0 = x(23500 - 1000x)$$

$$x = 0 \quad \text{or} \quad 23500 - 1000x = 0$$

$$x = \$23.50$$



c. Find the price that maximizes revenue from ticket sales.

$$x: \frac{-b}{2a} = \frac{-23500}{-2000} = \$11.75$$

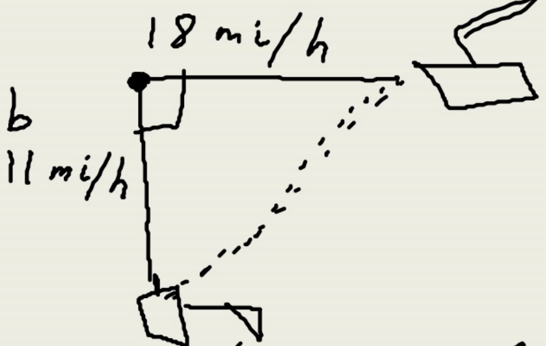
$$R(11.75) =$$

$3x \cdot 2x = 6x^2 \cdot \quad 3x + 2x = 5x$.

Two ships leave port at the same time. One sails south at 11mi/h and the other sails east at 18mi/h. Find a function that models the distance D between the ships in terms of the time t ~~(in hours) elapsed~~ since their departure. a

Pythagorean : $\sqrt{a^2+b^2} = \sqrt{c^2}$ hypodenuse.

$\sqrt{a^2+b^2} = c$



$$D(t) = \sqrt{(18t)^2 + (11t)^2}$$
$$= \sqrt{324t^2 + 121t^2}$$
$$= \sqrt{445t^2} = \boxed{\sqrt{445}\, t}.$$

$t=$
After 1 hr
$a = 18(1)$
$b = 11(1)$
$c = \sqrt{18^2 + 11^2}$

$t:$
After 2hrs.
$a = 36 = 18(2)$
$b = 22 = 11(2)$
$c = \sqrt{(18(2))^2 + (11(2))^2}$

$t=$
After 3hrs.
$a = 54 = 18(3)$
$b = 33 = 11(3)$
$c = \sqrt{(18(3))^2 + (11(3))^2}$

Find two positive numbers whose sum is 100 and the sum of whose squares is a minimum.

$$x + y = 100 \Rightarrow x + (100 - x) = 100$$

ex. $25 + 75 = 100$

$25 + (100 - 25) = 100$

$$x^2 + (100 - x)^2 = x^2 + 10000 - 200x + \ldots$$

$$2x^2 - 200x + 10000$$

$$x: \frac{-b}{2a} = \frac{200}{2(2)} = \frac{200}{4} = 50$$

50 and 50

Homework 9/22

Pg. 211-212 # 13-15, 21-23, 27.

~~TB pg. 210-211 #1, 3, 7, 9, 11, 13, 21, 25~~