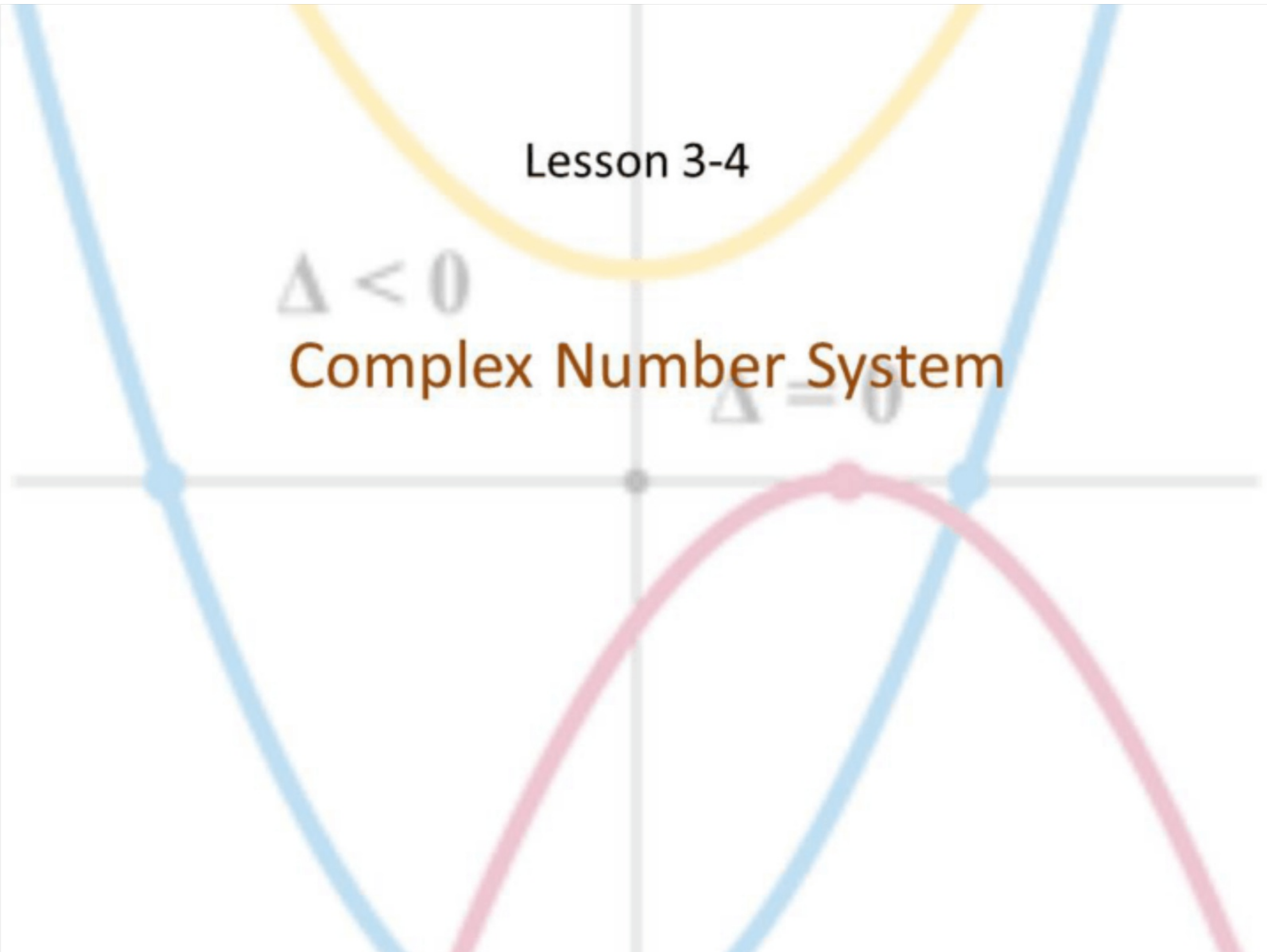
Lesson 3-4
Complex Number System
Δ < 0
Δ = 0

Objective

Students will…

- Be able to define the complex number system, built on the number $i = \sqrt{-1}$.
- Be able to perform arithmetic operations on complex numbers.

Square Root of Negative Numbers

We observed in the past that real numbers alone had some limitations when solving for certain quadratic equations. This was due to the fact that some quadratics required taking the square root of a negative number. For example, to find the zeros of the following polynomial,

$$P(x) = x^2 - x + 1$$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)} \qquad \text{Quadratic Formula}$$

$$x = \frac{1 \pm \sqrt{1-4}}{2} = \mathbf{\frac{1 \pm \sqrt{-3}}{2}}$$

In the past, we'd simply write, "no real solutions," for such equations. So, in order to solve all quadratic equations, mathematicians created an expanded number system called, *the Complex Number System*.

The Complex Number System

The complex number system is built entirely on the complex number "i" defined as $i = \sqrt{-1}$.

A complex number is a an expression of the form,

$a + bi$, where *a* and *b* are real numbers and $i^2 = -1$.

The real part of this complex number is *a* and the imaginary part (the number in front of the "i") is *b*. Two complex numbers are equal if and only if **both** their real and imaginary parts are equal.

Examples

The following are examples of complex numbers

$3 + 4i$	Real part 3, imaginary part 4 i
$\frac{1}{2} - \frac{2}{3}i$	Real part $\frac{1}{2}$, imaginary part $-\frac{2}{3}$ i
$0 + 6i$	Real part 0, imaginary part 6 i
$-7 + 0i$	Real part -7, imaginary part 0 i

$2\sqrt{3}$

Complex Solutions

$\sqrt{-2} = i\sqrt{2}$

$x \cdot 2 = 2x \quad x2 \neq x^2 \quad \sqrt{2}\,i$

So, with that said, we can find a solution to **<u>every</u>** quadratic equation.

$$Q(x) = x^2 + 9$$

ex. $\sqrt{-4} = 2i$

$$0 = x^2 + 9$$

$$-9 = x^2$$

$\sqrt{-16} = 4i$

$$\pm 3i = \underset{i}{\sqrt{-1}}\,\underset{3}{\sqrt{9}} = \sqrt{-9} = x$$

Using the complex number system, we can see that $\mathrm{Q}(x)$ has solutions (zeros) $\pm 3i$ because,

$$(3i)^2 \overset{=3^2i^2}{=} 9i^2 = 9(-1) = -9 \text{ and } \overset{(-3)^2i^2}{(-3i)^2} = 9i^2 = -9$$

So from here we can see that for any number a, $\sqrt{-a} = \sqrt{a}(i)$.

Operations on Complex Numbers

Like all other number systems, we need to be able to perform arithmetic operations on the complex number system. Fortunately, this isn't very difficult, because it's very similar to algebra (i.e treat i like a variable)

$3+2x+4+6x$ $\qquad$ $(3+4)+(2+6)x$

Add/Sub: $(a+bi) \pm (c+di) = (a+c) \pm (b+d)i$

Multiply: $(a+bi)\cdot(c+di) = (ac + a(di) + (bi)c + (bi)(di))$

$$= (ac + adi + cbi + bd(i^2)) \qquad \textbf{(FOIL)}$$

$$= (ac + adi + cbi + bd(-1))$$

$$= (ac + adi + cbi - bd)$$

$$= (ac - bd) + (ad + cb)i$$

$(x+2)(x-2) = x^2 - 4$

$\frac{3}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}}$

Dividing Complex Numbers with Conjugates

$i = \sqrt{-1}$

$i^2 = -1$

To simplify the quotient $\frac{a+bi}{c+di}$, we need to <u>multiply</u> the numerator and the denominator by the complex **<u>conjugate</u>** of the denominator:

$$\frac{a+bi}{c+di} = \left(\frac{a+bi}{c+di}\right)\left(\frac{c-di}{c-di}\right) = \frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$$

Example:

$$\frac{3+5i}{1-2i} = \left(\frac{3+5i}{1-2i}\right)\left(\frac{1+2i}{1+2i}\right) = \frac{3+6i+5i+10i^2}{1^2+2^2} = \frac{3+11i-10}{5} = \frac{-7+11i}{5}$$

Examples

$i^{26} = (i^2)^{13}$
$= (-1)^{13} = \boxed{-1}$

$i^{28} = (i^2)^{14} = 1$

$i^{25} = (i^2)^{12} i$
$= (-1)^{12} i = \boxed{i}$

1. $(3 + 5i) + (4 - 2i)$

$7 + 3i$

2. $(3 + 5i) - (4 - 2i)$

$-1 + 7i$

3. $(3 + 5i)(4 - 2i)$

$12 - 6i + 20i - 10i^2$ (-1)

$= 12 + 14i + 10 = \boxed{22 + 14i}$

4. $\frac{(7+3i)}{4i} \cdot \frac{-4i}{-4i}$ (Note: Conjugate of $4i = -4i$)

$= \frac{-28i - 12i^2}{-16i^2} = \frac{-28i + 12}{16}$

$= \boxed{\frac{-7i + 3}{4}}$

5. $i^{23} = (i^2)^{11} i = (-1)^{11} i$
$= -i$

Homework 10/23

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