Lesson 3-3

The First Derivative Test

Objective

Students will…

- Be able to determine intervals on which a function is increasing or decreasing.
- Be able to apply the First Derivative Test to find relative extrema of a function.

Increasing vs Decreasing

Recall from the past that…

A function f is **<u>increasing</u>** on an interval if for any two numbers a and b in the interval, $a < b$ implies $f(a) < f(b)$.

A function f is **<u>decreasing</u>** on an interval if for any two numbers a and b in the interval, $a < b$ implies $f(a) > f(b)$.

In other words, moving from **<u>left to right</u>**, if the graph is going up it is increasing, while if it goes down it is decreasing.

Derivatives and Inc/Dec

Considering that the derivative of a function is the equation that finds the rate of change of a function, we have this trivial result…

Theorem 3.5- Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). Then…

1. If $f'(x) > 0$, i.e. positive, for all x in (a, b), then f is increasing on $[a, b]$.
2. If $f'(x) < 0$, i.e. negative, for all x in (a, b), then f is decreasing on $[a, b]$.
3. If $f'(x) = 0$, for all x in (a, b), then f is constant on $[a, b]$.

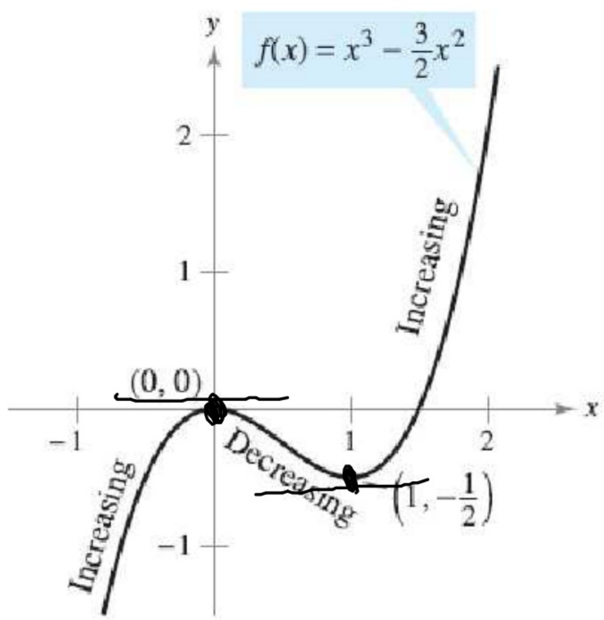
Remember, derivative represents the slope!

Example

Find the open intervals on which $f(x) = x^3 - \frac{3}{2}x^2$ is decreasing or increasing. (Graphically)

Inc: $(-1, 0) \cup (1, 2)$

Dec: $(0, 1)$



Example

Find the open intervals on which $f(x) = x^3 - \frac{3}{2}x^2$ is decreasing or increasing. (Algebraically)

$f'(x) = 3x^2 - 3x$

$0 = 3x^2 - 3x$

$0 = 3x(x-1)$

$x = 0, 1$

CP

-1	0	$\frac{1}{2}$	1	2
$+$		$-$		$+$

Inc: $(-\infty, 0) \cup (1, \infty)$

Dec: $(0, 1)$

Example

Find the open intervals on which $y = \frac{x^2}{x+2}$ is decreasing or increasing.

f

g

$$y' = \frac{2x(x+2) - x^2}{(x+2)^2}$$

$$0 = \frac{2x(x+2) - x^2}{(x+2)^2} = \frac{2x^2 + 4x - x^2}{(x+2)^2}$$

$$0 = \frac{\boxed{x^2 + 4x}}{(x+2)^2}$$

$$0 = x(x+4)$$

$$\boxed{x = 0, -4 \text{ CN}}$$

-5	-1	1
+	−	+

-4, 0

Inc: $(-\infty, -4) \cup (0, \infty)$

Dec: $(-4, -2) \cup (-2, 0)$

First Derivative Test

Putting all of this together, we come up with the **First Derivative Test**, which allows us to find all of the relative minimums and maximums.

minima maxima

(value)

The First Derivative Test- Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows.

1. If $f'(x)$ changes from **negative to positive** at c, then f has a **relative minimum** at $(c, f(c))$.
2. If $f'(x)$ changes from **positive to negative** at c, then f has a **relative maximum** at $(c, f(c))$.
3. If $f'(x)$ either positive or negative on both sides of c, then $f(c)$ neither a relative minimum nor a relative maximum.

Example

$C = CV$

Find the relative extrema of $f(x) = x^3 - \frac{3}{2}x^2$

	0	1	
-1		$\frac{1}{2}$	2
$+$		$-$	$+$

c	$f(c)$
rel. max @	$(0, 0)$
rel. min @	$(1, -\frac{1}{2})$

Example

Find the relative extrema of $y = \frac{x^2}{x+2}$

-5	-4	-1	0	1
$+$		$-$		$+$

rel. max @ $(-4, -8)$

rel. min @ $(0, 0)$

Example

$\pi/6 - \sqrt{3}/2$

Find the relative extrema of the function $f(x) = \frac{1}{2}x - \sin x$ $[0, 2\pi]$

$f'(x) = \frac{1}{2} - \cos x$

$0 = \frac{1}{2} - \cos x$

$\frac{1}{2} = \cos x$

$x = \pi/3,\ 5\pi/3$

0	$\pi/3$	π	$5\pi/3$	2π
$-$		$+$		$-$

rel min @ $(\pi/3,\ \pi/6 - \sqrt{3}$

rel max @ $(5\pi/3,\ 5\pi/6 + \sqrt{}$

Example

Find the relative extrema of $f(x) = (x^2 - 4)^{\frac{2}{3}}$

Example

Find the relative extrema of $f(x) = \frac{x^4+1}{x^2}$

Homework 10/26

3.3 #1-8, 9-15 (odd), 17-37 (e.o.o), 39-45 (odd)