Warm Up 10/14

For the following functions, find the zeros and describe their end behavior.

1. $P(x) = x^2 - 4$

$(x+2)(x-2)=0$

$x = \pm 2$

even +

$x \to \infty, y \to \infty$

$x \to -\infty, y \to \infty$

2. $Q(x) = -2(x+2)^4$

$x = -2$

even (−

$x \to \infty, y \to -$

$x \to -\infty, y \to$

3. $R(x) = -(x-2)^2(x+1)^3$

$x = 2, -1$

odd (−)

$x \to \infty \quad y \to -\infty$

$x \to -\infty \quad y \to \infty$

4. $S(x) = (x^3 + 3x^2) - (9x - 27)$ odd

$x^2(x+3) - 9(x+3)$

$(x^2-9)(x+3) = 0$

$(x+3)(x-3)(x+3) = 0$

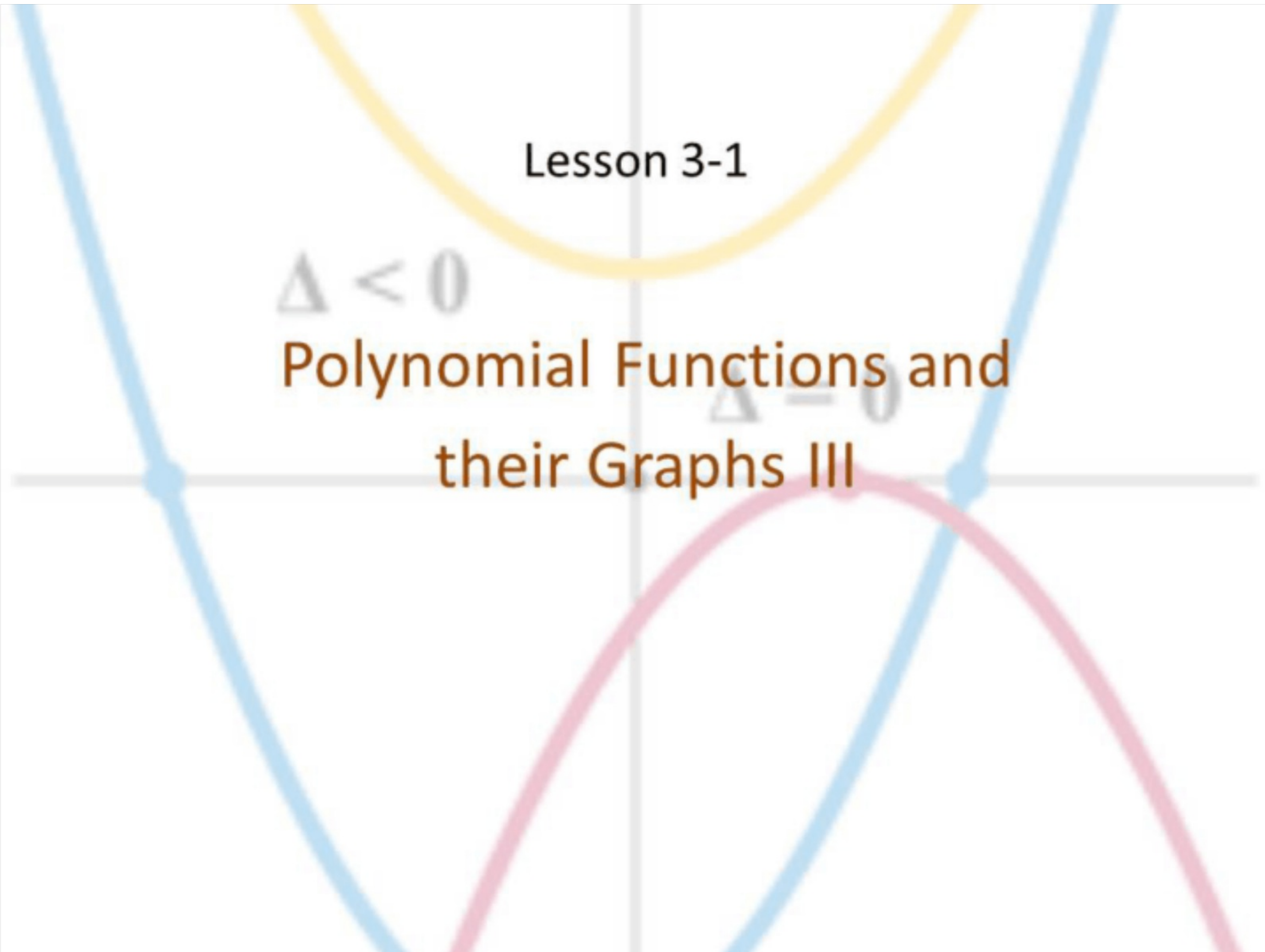
$x = -3, 3$

$x \to \infty,$

$x \to -\infty,$

Lesson 3-1

Polynomial Functions and their Graphs III



Objective

Students will…

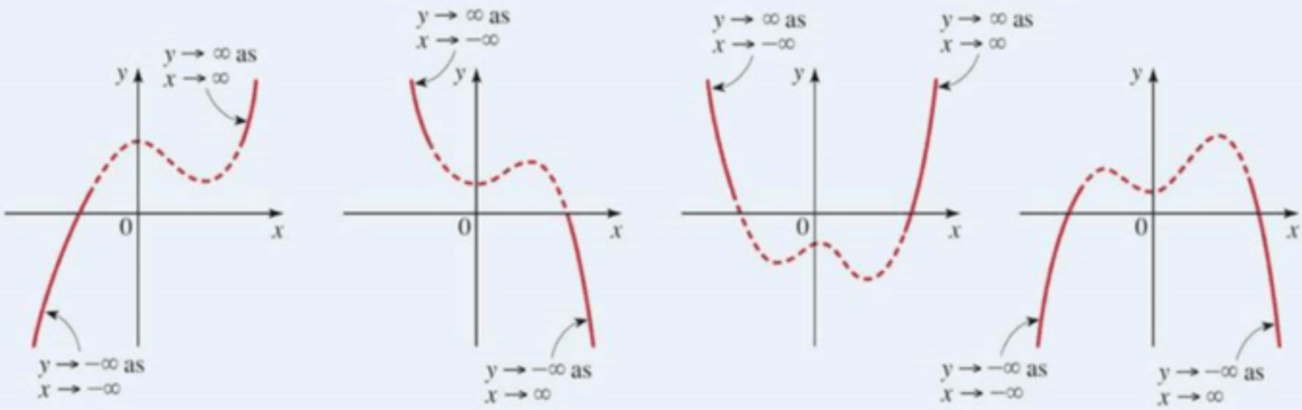
- Be able to find and apply the multiplicity of each zero to graph polynomial functions.

End Behavior

END BEHAVIOR OF POLYNOMIALS

The end behavior of the polynomial $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ is determined by the degree n and the sign of the leading coefficient a_n, as indicated in the following graphs.

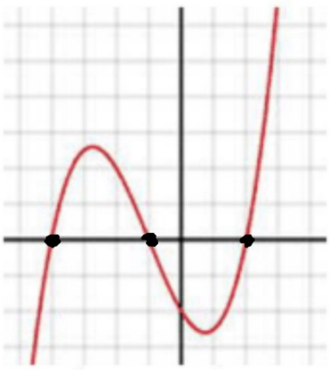
***P* has odd degree** | ***P* has even degree**



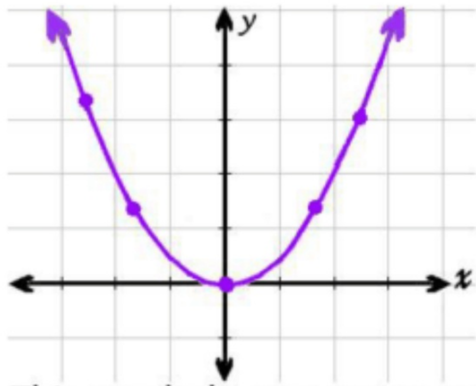
Leading coefficient positive | Leading coefficient negative | Leading coefficient positive | Leading coefficient negative

Shape of the Graph Near a Zero

As we can observe from various graphs, we see that some **cross** the x-axis, while some do not. For example,



This cubic graph crosses the x-axis 3 times.

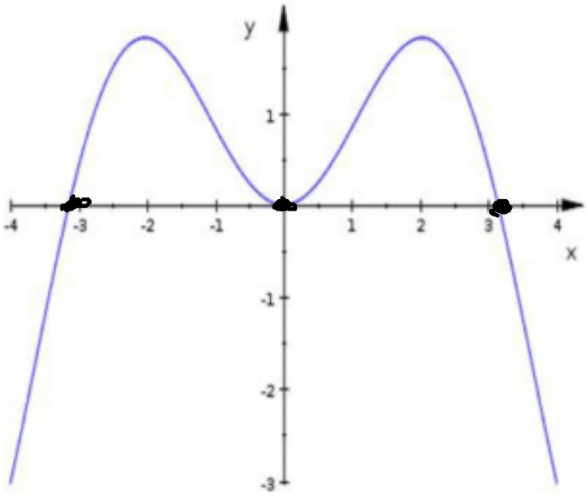


This parabola never crosses the x-axis.

Note: Touching and crossing are different things!

Shape of the Graph Near a Zero

In fact, some graphs contain a mixture of crossing and no crossing behavior. Consider,



Here, we can see that while the graph crosses the x-axis at the two ends, it does not cross the x-axis in the middle. (Note: the graph is still **touching** the x-axis in all places, so they are still considered as zeros of the graph).

Crossing Behavior

We can also observe a nice pattern regarding the x-axis crossing behaviors of polynomial graphs. The pattern has to do with the multiplicity of every zero, i.e. the exponent attached to them.

Example: $P(x) = x^4(x-2)^3(x+1)^2$

$x\,x\,x\,x\,(x-2)(x-2)(x-2)(x+1)(x+1)$

For the above polynomial function $P(x)$, its zeros are 0, 2, and -1. The multiplicity of these zeros are the exponents that are attached to each of them. So, the x-intercept 0 has the multiplicity of 4, while 2 has the multiplicity of 3, and -1 has the multiplicity of 2.

Multiplicity and the Crossing Behavior

With that said, the pattern regarding the x-axis crossing behavior is as follows

For every odd multiplicity, the graph at that particular x-intercept, will **cross** the x-axis.

For every even multiplicity, the graph at that particular x-intercept, will **not cross** the x-axis.

So from our previous example, since the x-intercepts 0 and -1 had an even multiplicity, the graph will not cross the x-axis at those points. In contrast, at the intercept 2, the graph will cross the x-axis because it had an odd multiplicity.

Graphing Polynomials

How is this useful? Well, understanding where the graph does and doesn't cross the x-axis will seriously aid in graphing the polynomials.

P(1) =

Example: Graph the polynomial, $P(x) = x^4(x-2)^3(x+1)^2$

mult: E O E

Zeros: 0, 2, -1

Y-intercept: (0,0)

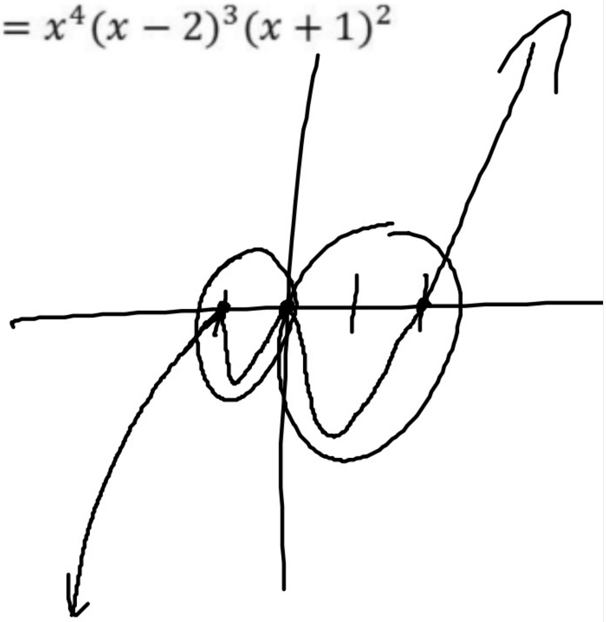
Degree: 9 (odd)

+ or - ?: +

End Behavior:

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$



Example

1 0 -2 -125

Graph the polynomial: $R(x) = (x-1)^2(x-2)(x-5)^3$

(O or E)

mult: E O O

Zeros: 1, 2, 5

Y-intercept: (0, 250)

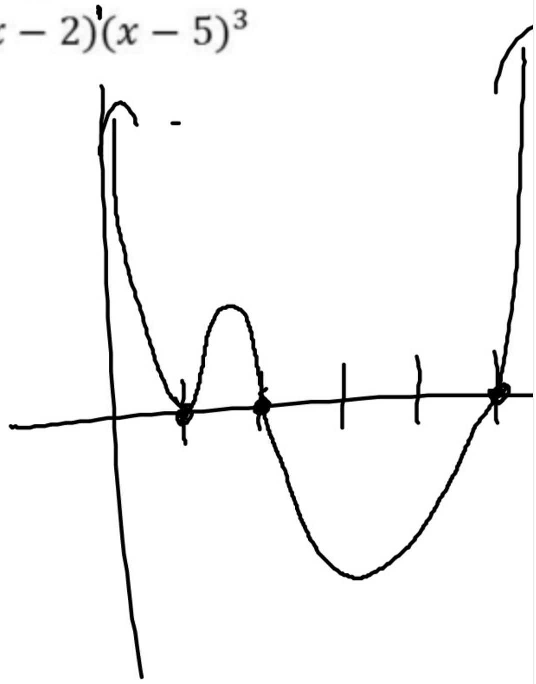
Degree: 6 (even)

+ or - ?: +

End Behavior:

$x \to \infty, y \to \infty$

$x \to -\infty, y \to \infty$



Example

Graph the polynomial: $T(x) = -x^4 + 3x^3 - 2x^2$

$0 = -x^2(x^2 - 3x + 2) = -x^2(x-2)(x-1)$

mult: e o o

Zeros: 0, 2, 1

Y-intercept: (0, 0)

Degree: 4 (even)

\+ or - ?: −

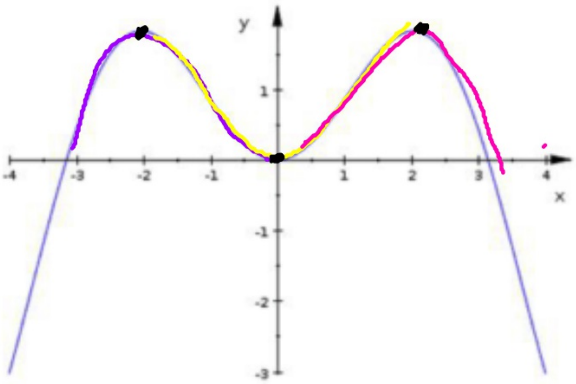
End Behavior:

$x \to \infty,\ y \to -\infty$

$x \to -\infty,\ y \to -\infty$

Local Extrema (Maxima and Minima)

The last thing to observe in this section is the local extrema. Extrema refers to both maxima and minima of a graph. For Example,



We can see that this graph contains three local extrema.

Local Extrema Principle

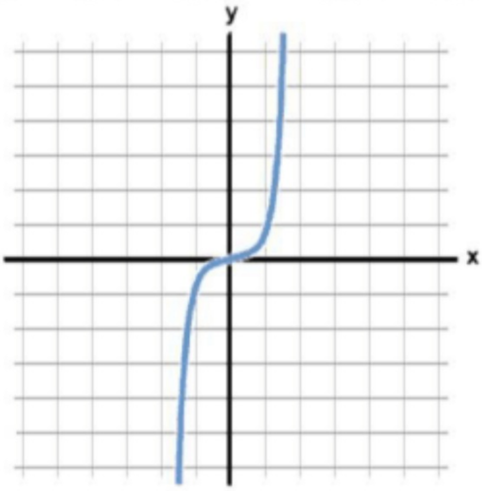
The Local Extrema Principle states that for every polynomial of degree, n, the graph has **at most** $n - 1$ local extrema.

So for example, $\mathrm{T}(x) = -x^4 + 3x^3 - 2x^2$, can have no more than $4 - 1 = 3$ local extrema. We can see this on our graph.

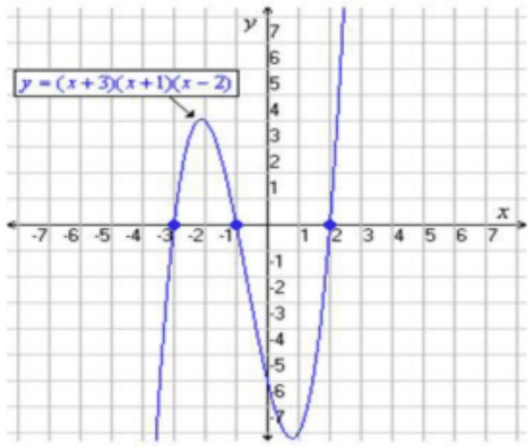
Also, $P(x) = x^4(x-2)^3(x+1)^2$, can have no more than $9 - 1 = 8$ local extrema. We can also see this on our graph.

The key word here is of course, "**at most**." Hence, a polynomial can have less than $n-1$ local extrema. In fact, standard function $P(x) = x^3$ does not have any local extrema, while

$y = (x+3)(x+1)(x-2)$ has exactly 2 (which is okay since 3-1=2).



$P(x) = x^3$



$y = (x+3)(x+1)(x-2)$

Homework 10/14

TB pg. 262 #11-35 (e.o.o)

Use the zeros, end behaviors, and the multiplicity to sketch the graph.