Lesson 3-1

Extrema On An Interval

Objective

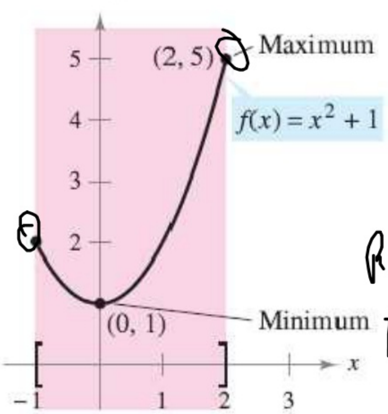
Students will…

- Be able to understand what an extrema is over an open and closed intervals.
- Be able to distinguish between relative and an absolute extrema.

Extrema

One of the big applications of differentiation is finding the extrema (plural form of the word extremum) of functions. Extrema are the maximum and minimum (extreme) values of a function over an interval.

Extreme Value Theorem- If f is continuous on a closed interval $[a, b]$, then f has both a minimum and a maximum on the interval.



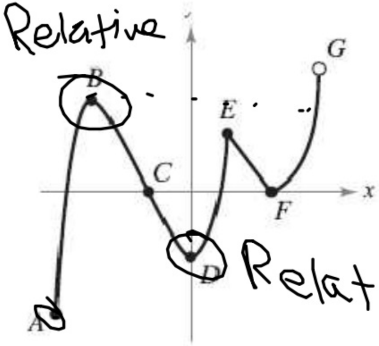
$(-1, 2)$

Relative
Absolute.

highest/ lowest. → # Absolute vs Relative Extrema ← Non-endpoint highest/lowest.

A great way to distinguish absolute and relative extrema is to consider whether the interval is open or closed.

Relative



Relative

On an open interval (A, G), there are no single extreme values.

On a closed interval $[A, G]$, however, there are single extreme values. Absolute.

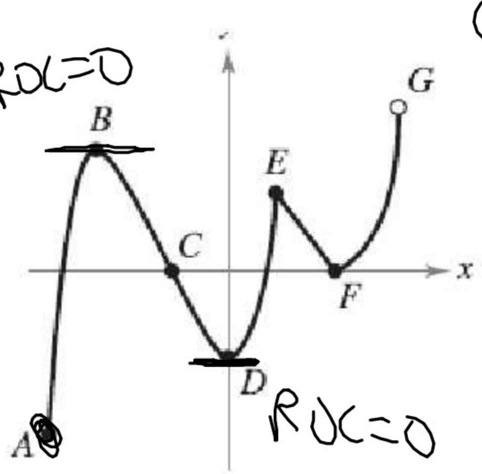
Another good way to identify relative extrema are to think of relative maximum as a hill (or a mountaintop), and relative minimum as a valley.

Application of Differentiation

What connection can we make between derivatives and the extrema?

(A, G)

ROC=0



ROC=0

Relative Extrema can only be points where its instantaneous rate of change is 0.

Relative extrema of f(x) can only be x values that satisfy the equation $f'(x)=0$.

These solutions are called "Critical Values".

Application of Differentiation

Overall, we can formulize the steps in finding the extrema.

GUIDELINES FOR FINDING EXTREMA ON A CLOSED INTERVAL

To find the extrema of a continuous function f on a closed interval $[a, b]$, use the following steps.

1. Find the critical numbers of f in (a, b).
2. Evaluate f at each critical number in (a, b).
3. Evaluate f at each endpoint of $[a, b]$.
4. The least of these values is the minimum. The greatest is the maximum.

For open intervals, only do steps 1 and 2.

Example

Find the extrema of $f(x) = 3x^4 - 4x^3$ on the interval $[1, 2]$.

$f'(x) = 12x^3 - 12x^2$

$\Rightarrow 0 = 12x^3 - 12x^2$

$0 = 12x^2(x-1)$

$x = 1, 0$

C.V

$f(0) = 0$

$f(1) = 3 - 4 = -1$

$f(2) = 48 - 32 = 16$

Absolute. min $= -1$

max $= 16$

Example

~~$3 \cdot 2/3$~~

Find the extrema of $f(x) = 2x - 3x^{\frac{2}{3}}$ on the interval $[-1, 3]$.

$f'(x) = 2 - 2x^{-1/3}$

$\Rightarrow 0 = 2 - 2x^{-1/3}$

$\frac{0}{2} = \frac{\cancel{2}(1 - x^{-1/3})}{\cancel{2}}$

$0 = 1 - x^{-1/3}$

$x^{-1/3} = 1 \quad x = 1$

$f(-1) = -2 - 3 = -5$ min

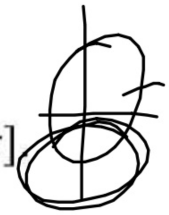
$f(1) = 2 - 3 = -1$

$f(3) = 6 - 3\sqrt[3]{9}$ max

$\frac{7}{3} > \sqrt[3]{9} > \sqrt[3]{8} = 2$

Example

Find the extrema of $f(x) = 2\sin x - \cos(2x)$ on the interval $[0, 2\pi]$.



$$f'(x) = 2\cos x + \sin 2x \cdot (2)$$

$$0 = 2\cos x + 2\sin 2x$$

$$0 = \cancel{2}(\cos x + \sin 2x)$$

$$\cos x = -\sin 2x$$

$$\frac{\cancel{\cos x}}{\cancel{\cos x}} = \frac{-2\sin x \cancel{\cos x}}{\cancel{\cos x}}$$

$$\frac{1}{-2} = \frac{\cancel{-2}\sin x}{\cancel{-2}}$$

$$\sin x = -\tfrac{1}{2}$$

$$x = \tfrac{7\pi}{6}, \tfrac{11\pi}{6}, 0, 2\pi$$

Homework 10/23

3.1 Ex #1-2, 3-11 (odd), 13-35 (odd), 37, 39