Crossing Behavior

We can also observe a nice pattern regarding the x-axis crossing behaviors of polynomial graphs. The pattern has to do with the multiplicity of every zero, i.e. the exponent attached to them.

$= xxxx(x-2)(x-2)(x-2)(x+1)(x+1)$

Example: $P(x) = x^4(x-2)^3(x+1)^2$

$x = 0, 2, -1$

For the above polynomial function $P(x)$, its zeros are 0, 2, and -1. The multiplicity of these zeros are the exponents that are attached to each of them. So, the x-intercept 0 has the multiplicity of 4, while 2 has the multiplicity of 3, and -1 has the multiplicity of 2.

Multiplicity and the Crossing Behavior

With that said, the pattern regarding the x-axis crossing behavior is as follows

For every <u>odd multiplicity</u>, the graph at that particular x-intercept, will **cross** the x-axis.

For every <u>even multiplicity</u>, the graph at that particular x-intercept, will **not cross** the x-axis.

So from our previous example, since the x-intercepts 0 and -1 had an even multiplicity, the graph will not cross the x-axis at those points. In contrast, at the intercept 2, the graph will cross the x-axis because it had an odd multiplicity.

Graphing Polynomials

x^2+2x+1
$(x+1)(x+1)$

How is this useful? Well, understanding where the graph does and doesn't cross the x-axis will seriously aid in graphing the polynomials.

$1(-8)(4)$

Example: Graph the polynomial, $P(x) = x^4(x-2)^3(x+1)^2$

E o E

Zeros: $0, 2, -1$

Y-intercept: $(0,0)$

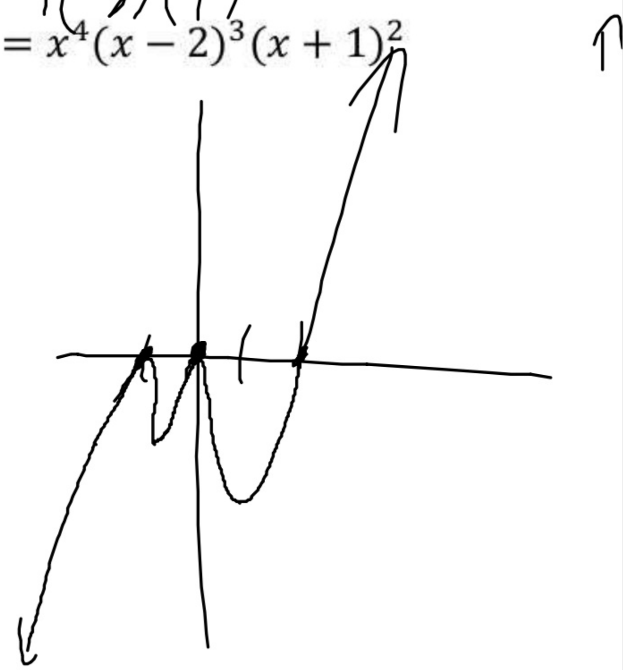
Degree: 9 (odd)

+ or - ?: +

End Behavior:

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$



Example

Graph the polynomial: $R(x) = (x-1)^2(x-2)(x-5)^3$

E O O

Zeros: 1, 2, 5

Y-intercept: (0, +250)

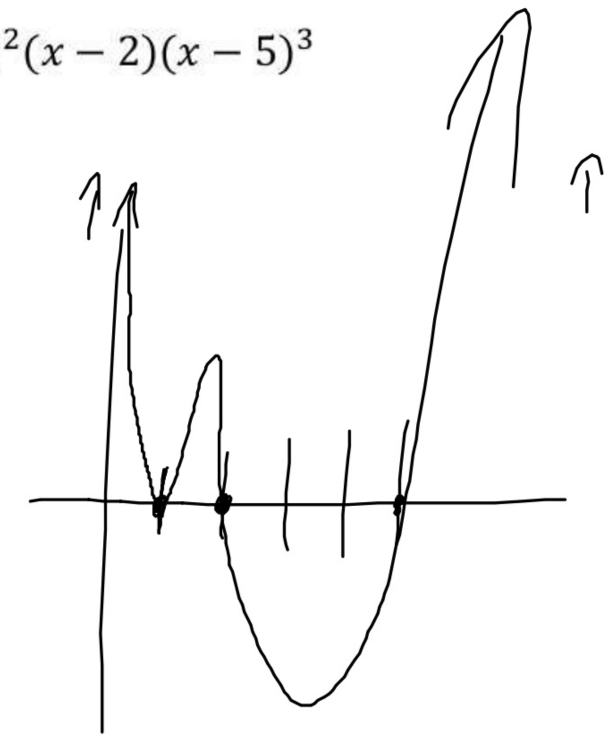
Degree: 6 (Even)

+ or - ?: +

End Behavior:

$x \to \infty$, $y \to \infty$

$x \to -\infty$, $y \to \infty$



Example

Graph the polynomial: $T(x) = -x^4 + 3x^3 - 2x^2 = -x^2(x^2 - 3x + 2)$

$(x-1)(x-2)$

Zeros: $\overset{E}{0}, \overset{O}{1}, \overset{O}{2}$

Y-intercept: $(0, 0)$

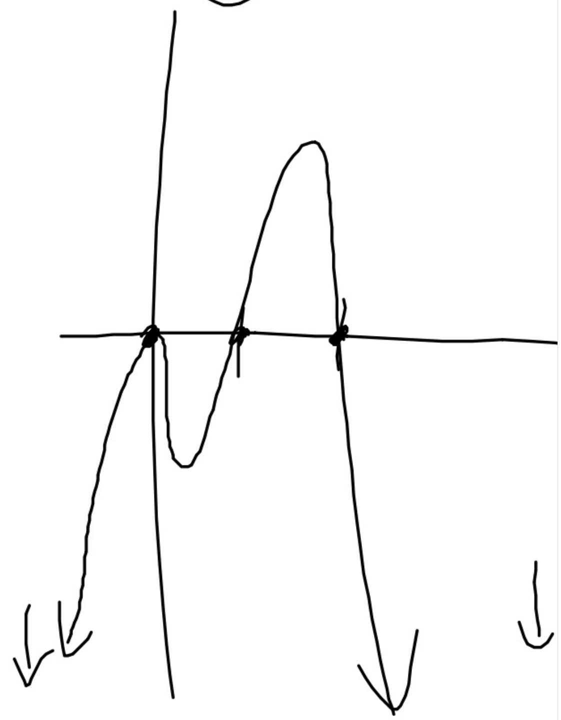
Degree: 4 (Even)

\+ or - ?: $(-)$

End Behavior:

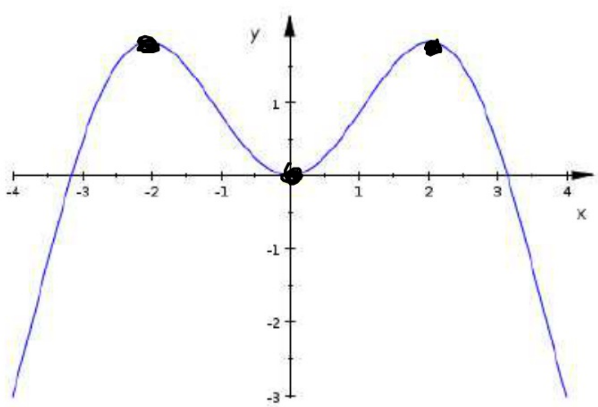
$x \to \infty, \; y \to -\infty$

$x \to -\infty, \; y \to -\infty$



Local Extrema (Maxima and Minima)

The last thing to observe in this section is the <u>local extrema</u>. Extrema refers to both maxima and minima of a graph. For Example,



We can see that this graph contains three local extrema.