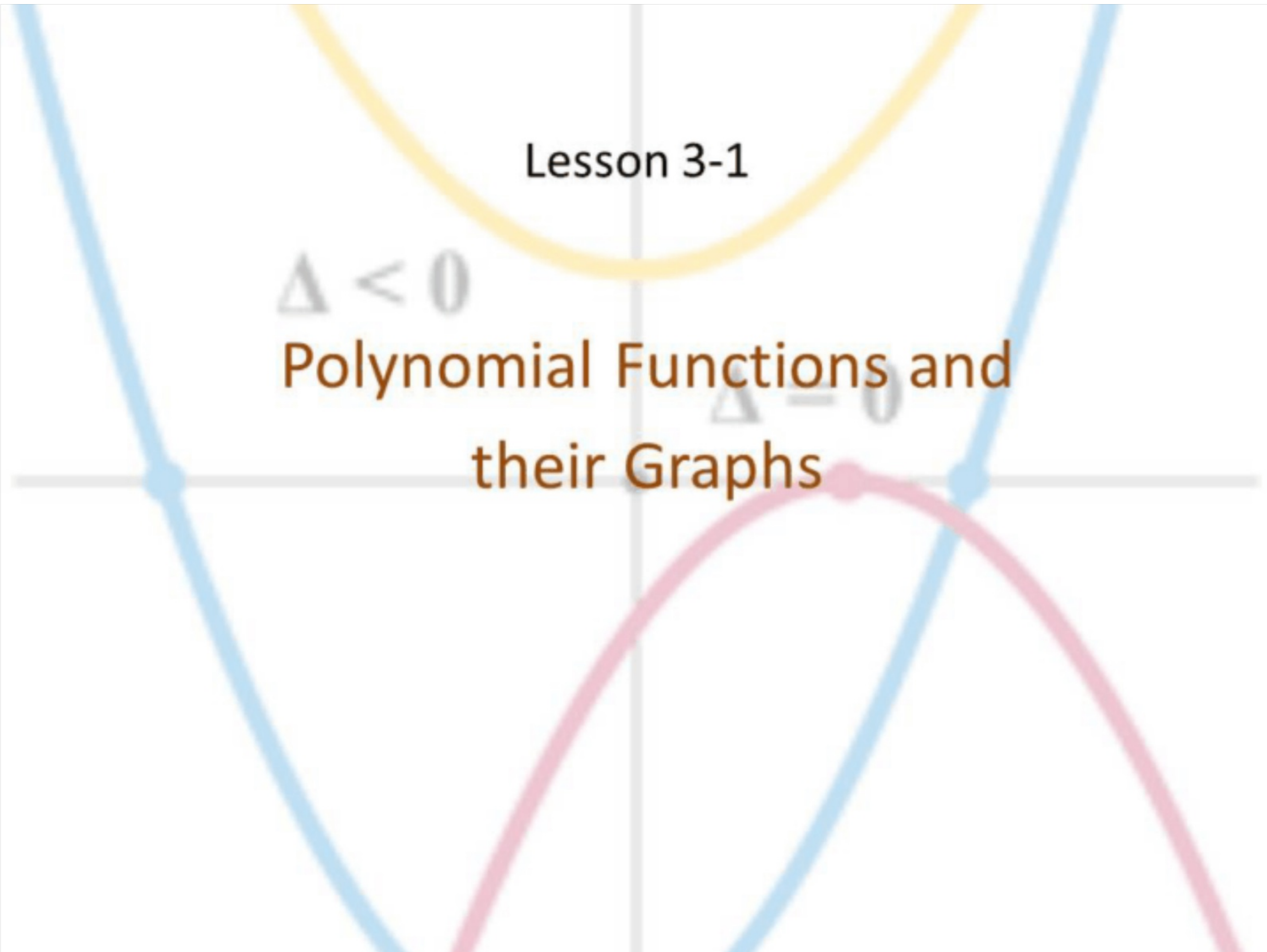
Lesson 3-1

Polynomial Functions and their Graphs



Objective

Students will…

- Be able to define and identify the characteristics of polynomials.
- Be able to find the x (zeros) and the y intercepts of polynomials by factoring, grouping, and using the quadratic formula.

$2x^2 + 3x^1 + 4x^0$

Polynomial Functions

$1 - 2x + 3x^2$

A polynomial function consists of polynomials, which take the form

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_1 x^1 + a_0 x^0,$$

where *n* is a nonnegative integer and $a_n \neq 0$.

The numbers $a_1, a_2, \ldots, a_n$ are called the coefficients.

The number a_0 is the constant coefficient or constant term.

The number a_n, the coefficient of the highest power, is the leading coefficient, and the term $a_n x^n$ is the leading term.

Example

Underline each coefficient, circle the constant term (coefficient), and box the leading term of the following polynomial function.

$$P(x) = \boxed{3x^5} + 6x^4 - 2x^3 + 1x^2 + 7x - 6$$

The function *P(x)* above is a polynomial of degree 5.

Polynomials

Here are other examples of different polynomials. Identify the degree of each polynomial.

	Degree
$P(x) = 3x^0$	0
$Q(x) = 4x - 7$	1
$R(x) = x^2 + x$	2
$S(x) = 2x^3 - 6x^2 - 10$	3

Polynomials with just a single term like *P(x)* is called a monomial.

Finding X, Y Intercepts

Finding the x and the y intercepts is an important step in analyzing polynomials. We will also use them for graphing in our next lesson.

To find y-intercept, we set $x = 0$ and find *y*.

To find x-intercept, we set $y = 0$ or $\mathrm{P}(x) = 0$ and find *x*.

Ex. Find the x and the y intercepts of $f(x) = 2x^2 - 1$

y-int: $f(0) = 0 - 1 = -1$

x-int: $0 = 2x^2 - 1$

$+1 \qquad +1$

$1 = 2x^2$

$\sqrt{\tfrac{1}{2}} = \sqrt{x^2} \Rightarrow x = \pm\sqrt{\tfrac{1}{2}}$

X-intercepts

$x^2 - 5x + 6$

$x = 2$ is a solution.

$(x-2)$

As we studied back in Algebra, there's a lot more to x-intercepts. We've learned that the x-intercepts are also known as roots or zeros of the function. All in all, the following are equivalent.

$+6$

$-3 \quad -2$

-5

$(x-3)(x-2)$

$x = 2, 3$

1. x-intercepts
2. Zeros or roots
3. Solutions to the polynomial equations
4. If c is the zero of a polynomial, then $(x - c)$ is one of its factors.

With that said, when you are instructed to find real zeros of a function, you are to find the x-intercepts.

Examples

Find the zeros of the following polynomials.

1. $P(x) = (x-2)(x+3)$

$0 = (x-2)(x+3)$

$x = 2, -3$

2. $Q(x) = (x+2)(x-1)(x-3)$

$x = -2, 1, 3$

3. $R(x) = x^3 - 2x^2 - 3x$

$0 = x^3 - 2x^2 - 3x$

$0 = x(x^2 - 2x - 3)$

$0 = x(x-3)(x+1)$

$\boxed{x = 0, 3, -1}$

4. $P(x) = -2x^3 - x^2 + x$

$0 = -x(2x^2 + x - 1)$

$= -x(2x-1)(x+1)$

$= \boxed{x = 0, 1/2, -1}$

5. $Q(x) = (x^3 + 3x^2) + (-4x - 12)$

6. $\cancel{R(x)}\ 0 = \frac{1}{8}(2x^4 + 3x^3 - 16x - 24)^2$

$0 = (2x^4 + 3x^3 - 16x - 24)^2$

$0 = (2x^4 + 3x^3)(-16x - 24)$

7. $S(x) = x^4 - 3x^2 - 4$

8. $Q(x) = 7b^2 - 7b + 10$

9. $R(x) = 2x^2 - 4x - 11$

Homework 10/10

Zeros of Polynomial WKSHT