Lesson 3-1

Polynomial Functions and their Graphs II

Objective

Students will…

- Be able to find the zeros and describe the end behaviors of polynomials.

Polynomial Functions

A polynomial function consists of polynomials, which take the form

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x^1 + a_0 x^0,$$

where *n* is a nonnegative integer and $a_n \neq 0$.

The numbers $a_1, a_2, \ldots, a_n$ are called the <u>coefficients</u>.

The number a_0 is the <u>constant coefficient</u> or <u>constant term</u>.

The number a_n, the coefficient of the highest power, is the <u>leading coefficient</u>, and the term $a_n x^n$ is the <u>leading term</u>.

Graphs of Polynomials

$y = 3x^0 = 3$

We already know that the graphs of polynomials of degree 0 or 1 are lines, and the graphs of polynomials of degree 2 are parabolas. The pattern is that the greater the degree of polynomials, the more complicated its graph can be.

However, there are two distinct natures of every polynomial graph.

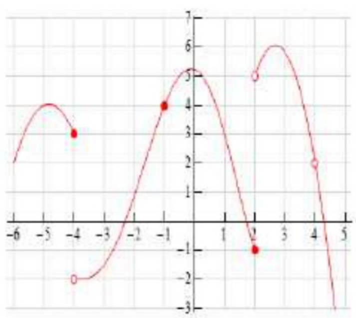
- Polynomial graphs are always smooth and curvy, i.e. no sharp corners

- Polynomial graphs are <u>continuous</u>, i.e. no breaks in between.

Example

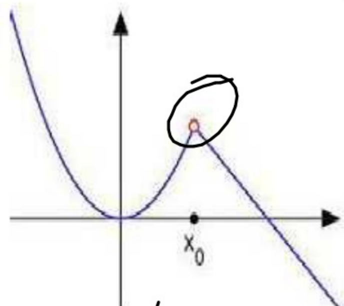
Determine whether following graphs are polynomial functions.

1.



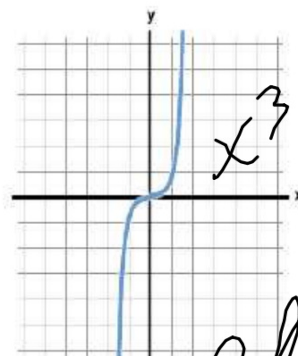
NO

2.



NO

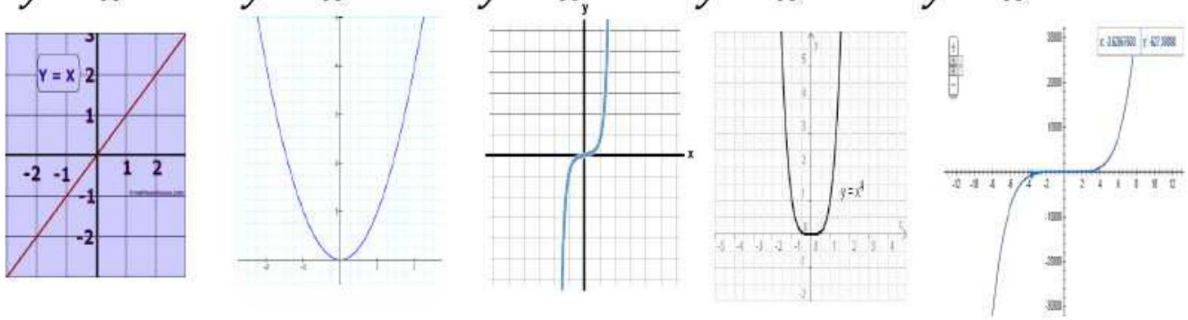
3.

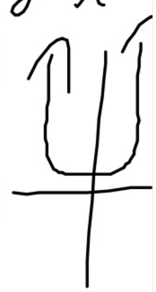


yes

The simplest polynomial functions are the monomials $P(x) = x^n$, whose graphs are often referred to as the standard graph. We have already seen standard graphs of linear and quadratics.

$y = x$ $\qquad y = x^2$ $\qquad y = x^3$ $\qquad y = x^4$ $\qquad y = x^5$ $\qquad y = x^6$





As you can see, as degree becomes larger, the graphs become flatter around origin and steeper elsewhere. We can see that all the odd number degree graphs are a variation of $y = x$, while all the even number degree graphs are a variation of $y = x^2$.

End Behavior

We saw a few distinct natures and patterns involving polynomial graphs. There is also a pattern regarding their end behavior.

Notation: approaching

$x \to \infty$ means "*x* becomes large in the positive direction"

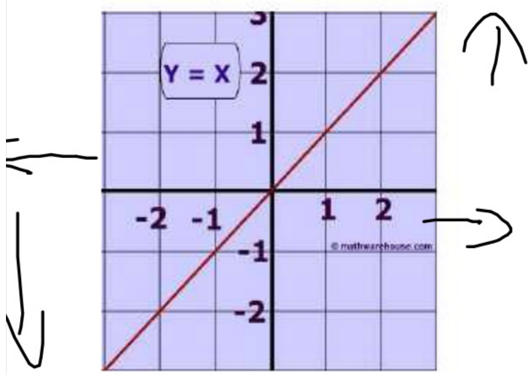
$x \to -\infty$ means "*x* becomes large in the negative direction"

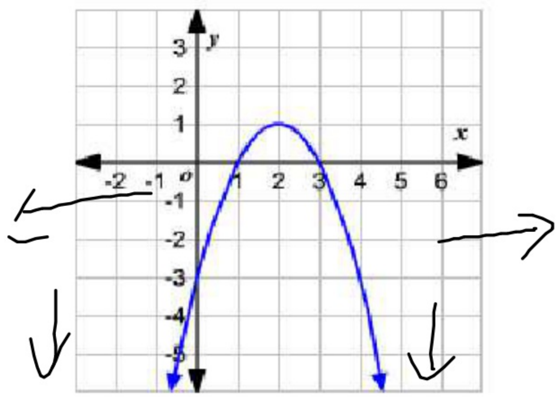
$y \to \infty$ means "*y* becomes large in the positive direction"

$y \to -\infty$ means "*y* becomes large in the negative direction"

End Behavior

For example, consider the following graph.





For $y = x$, $y \to \infty$ as $x \to \infty$,
and $y \to -\infty$ as $x \to -\infty$

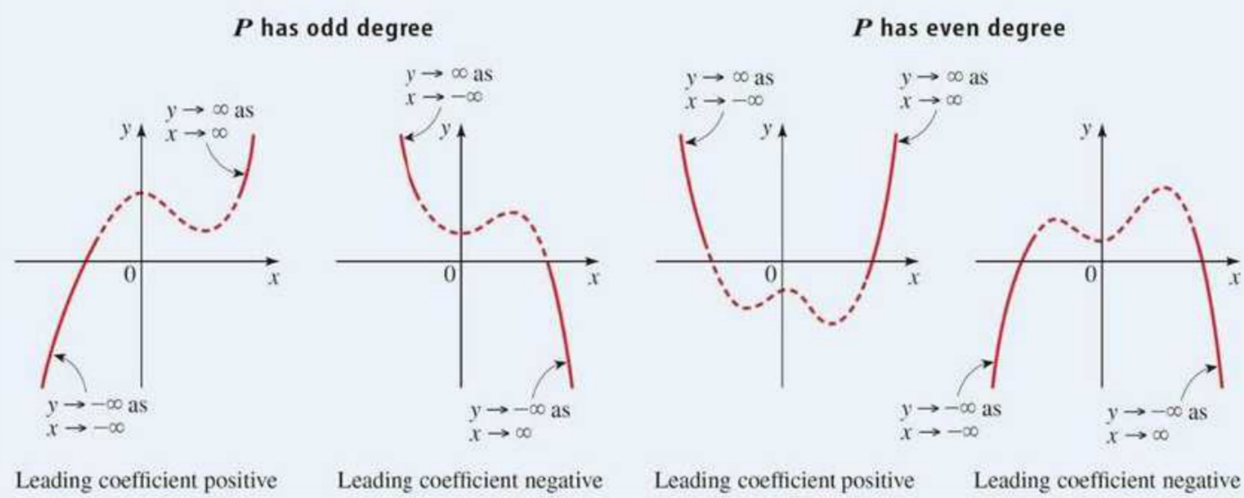
For $y = -x^2$, $y \to -\infty$, as $x \to \infty$,
as well as $x \to -\infty$

End Behavior

So, we can conclude that polynomials take the following patterns.

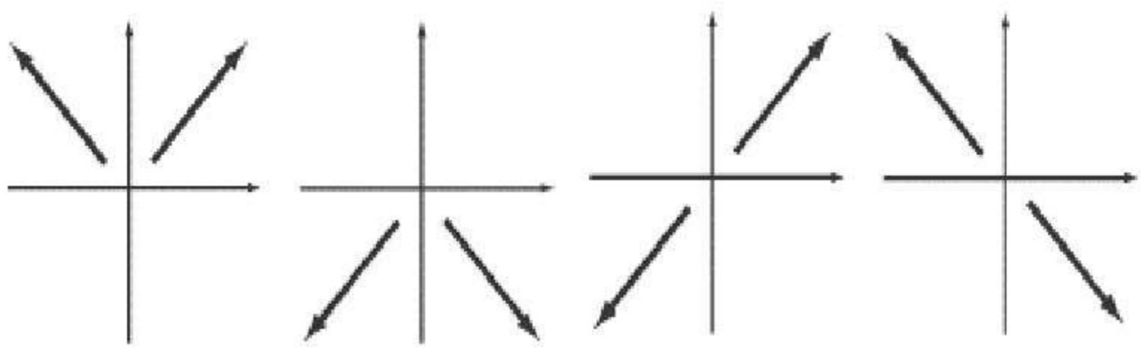
END BEHAVIOR OF POLYNOMIALS

The end behavior of the polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ is determined by the degree n and the sign of the leading coefficient a_n, as indicated in the following graphs.



End Behavior

So, we can conclude that polynomials take the following patterns.



n is even	n is even	n is odd	n is odd
$a_n > 0$	$a_n < 0$	$a_n > 0$	$a_n < 0$

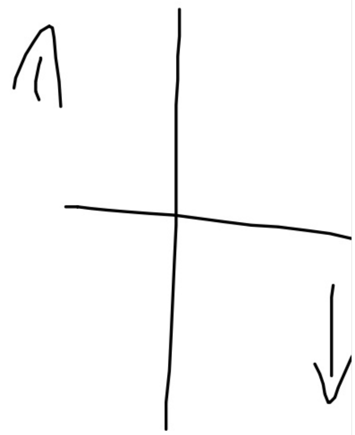
Examples

Find the real zeros of the following polynomials and describe their end behaviors.

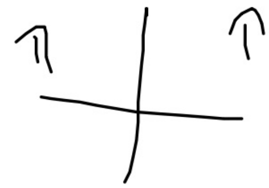
1. $P(x) = -x^3$ odd deg. $(-)$

$x = 0$

$$\boxed{\begin{array}{l} x \to \infty,\ y \to -\infty \\ x \to -\infty,\ y \to \infty \end{array}}$$



2. $Q(x) = (x-2)^4$ even deg. (+)



$x = 2$

$x \to \infty$, $y \to \infty$

$x \to -\infty$, $y \to \infty$

3. $R(x) = -2x^5 + 4$

odd deg. (−)

$\sqrt[5]{2} = \sqrt[5]{x^5}$

$\sqrt[5]{2} = x$

$x \to \infty$, $y \to -\infty$

$x \to -\infty$, $y \to \infty$

4. $P(x) = -2x^3 - x^2 + x$

5. $P(x) = (x^3 + 3x^2) + (-4x - 12)$

$0 = (x+3)(x^2-4)$

$x = -3, \pm 2$

6. $T(x) = x^6 - 2x^3 + 1$

$0 = (x^3-1)(x^3-1)$

$x = 1$

Odd deg. (+)

$x \to \infty, \ y \to \infty$

$x \to -\infty, \ y \to -\infty$

Even deg. (+)

$x \to \infty, \ y \to \infty$

$x \to -\infty, \ y \to \infty$

Homework 10/13

TB pg. 262 #11-35 (e.o.o) Just find the real zeros and describe end behaviors