Lesson 3-1

Polynomial Functions and their Graphs

Objective

Students will…

- Be able to define and identify the characteristics of polynomials.
- Be able to find the x (zeros) and the y intercepts of polynomials by factoring, grouping, and using the quadratic formula.

Polynomial Functions

A polynomial function consists of polynomials, which take the form

$3x^3 + 4x^1 + 1 \equiv 3x^3 + 0x^2 + 4x + 1x^0$

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_1 x^1 + a_0 x^0$$

$1 - 3x + x^2$

where *n* is a nonnegative integer and $a_n \neq 0$.

The numbers $a_1, a_2, \ldots, a_n$ are called the <u>coefficients</u>.

The number a_0 is the <u>constant coefficient</u> or <u>constant term</u>.

The number a_n, the coefficient of the highest power, is the <u>leading coefficient</u>, and the term $a_n x^n$ is the <u>leading term</u>.

Example

Underline each coefficient, circle the constant term (coefficient), and box the leading term of the following polynomial function.

$$P(x) = \boxed{3x^5} + 6x^4 - 2x^3 + 1x^2 + 7x - 6$$

The function *P(x)* above is a polynomial of degree 5.

Polynomials

Here are other examples of different polynomials. Identify the degree of each polynomial.

Degree

$P(x) = 3$ 0

$Q(x) = 4x - 7$ 1

$R(x) = x^2 + x$ 2

$S(x) = 2x^3 - 6x^2 - 10$ 3

Polynomials with just a single term like *P(x)* is called a monomial.

Finding X, Y Intercepts

Finding the x and the y intercepts is an important step in analyzing polynomials. We will also use them for graphing in our next lesson.

$\overset{x}{(0}, \overset{y}{a)}$

To find y-intercept, we set $x = 0$ and find *y*.

To find x-intercept, we set $y = 0$ or $\mathrm{P}(x) = 0$ and find *x*.

$\underset{x}{(a}, \underset{y}{0)}$

Ex. Find the x and the y intercepts of $f(x) = 2x^2 - 1$

y-int.

$f(0) = 2(0)^2 - 1$

$= -1$

$\boxed{(0, -1)}$

X-int.

$0 = 2x^2 - 1$

$1 = 2x^2$

$\frac{1}{2} = x^2$

$x = \pm\sqrt{\frac{1}{2}} \Rightarrow x = \pm\sqrt{1/2} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\left(\sqrt{\frac{1}{2}}, 0\right)$ $\left(-\sqrt{\frac{1}{2}}, 0\right)$

$\left(\pm\sqrt{1/2}, 0\right)$

X-intercepts

As we studied back in Algebra, there's a lot more to x-intercepts. We've learned that the x-intercepts are also known as <u>roots</u> or <u>zeros</u> of the function. All in all, the following are equivalent.

(a, 0)

1. x-intercepts
2. Zeros or roots
3. Solutions to the polynomial equations
4. If c is the zero of a polynomial, then $(x - c)$ is one of its factors.

With that said, when you are instructed to find real zeros of a function, you are to find the x-intercepts.

Examples

Find the zeros of the following polynomials.

$x-2=0$ or $x+3=0$

1. $P(x) = (x-2)(x+3)$

$\boxed{x=2, -3}$

2. $Q(x) = (x+2)(x-1)(x-3)$

$x = -2, 1, 3$

3. $R(x) = x^3 - 2x^2 - 3x$

$0 = x(x^2 - 2x - 3)$

$0 = x(x-3)(x+1)$

-3, -2

$x = 0, 3, -1$

4. $P(x) = -2x^3 - x^2 + x$

$0 = -x(2x^2 + x - 1)$

$0 = -x(2x-1)(x+1)$

$\boxed{x = 0, 1/2, -1}$

5. $Q(x) = x^3 + 3x^2 - 4x - 12$

a b c b

$x^2(x+3) - 4(x+3)$

$\rightarrow (x+3)(x^2-4)$

$0 = (x+3)(x+2)(x-2)$

$x = -3, \pm 2$

$\sqrt[3]{x^3} = \sqrt[3]{8}$

6. $R(x) = \frac{1}{8}(2x^4 + 3x^3 - 16x - 24)^2$

(8) (8)

$0 = (2x^4 + 3x^3 - 16x - 24)$

$0 = x^3(2x+3) - 8(2x+3)$

$\rightarrow 0 = (2x+3)(x^3-8)$

$\boxed{x = -3/2, 2}$

7. $S(x) = x^4 - 3x^2 - 4$

-4, -4, 1, -3

$0 = (x^2-4)(x^2+1)$

$= (x+2)(x-2)(x^2+1)$

$x = 2, -2$

$\sqrt{x^2} = \sqrt{-1}$

8. $Q(x) = 7b^2 - 7b + 10$

9. $R(x) = 2x^2 - 4x - 11$

Homework 10/10

Zeros of Polynomial WKSHT