Lesson 3-3

The First Derivative Test

Objective

Students will…

- Be able to determine intervals on which a function is increasing or decreasing.
- Be able to apply the First Derivative Test to find relative extrema of a function.

Increasing vs Decreasing

Recall from the past that…

A function f is **increasing** on an interval if for any two numbers a and b in the interval, $a < b$ implies $f(a) < f(b)$.

A function f is **decreasing** on an interval if for any two numbers a and b in the interval, $a < b$ implies $f(a) > f(b)$.

In other words, moving from **left to right**, if the graph is going up it is increasing, while if it goes down it is decreasing.

Derivatives and Inc/Dec

Considering that the derivative of a function is the equation that finds the rate of change of a function, we have this trivial result…

Theorem 3.5- Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). Then…

positive

1. If $f'(x) > 0$, i.e. positive, for all x in (a, b), then f is increasing on $[a, b]$.
2. If $f'(x) < 0$, i.e. negative, for all x in (a, b), then f is decreasing on $[a, b]$.
3. If $f'(x) = 0$, for all x in (a, b), then f is constant on $[a, b]$.

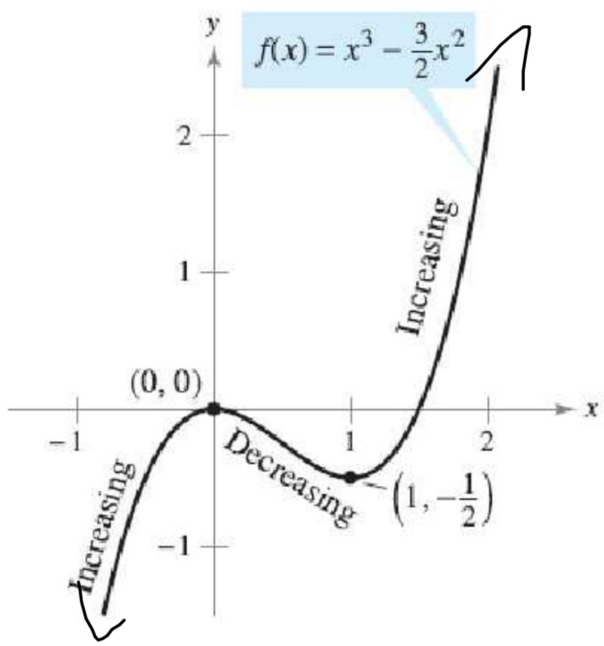
Remember, derivative represents the slope!

Example

Find the open intervals on which $f(x) = x^3 - \frac{3}{2}x^2$ is decreasing or increasing. (Graphically)

Inc: $(-\infty, 0) \cup (1, \infty)$

Dec: $(0, 1)$.



Example

Find the open intervals on which $f(x) = x^3 - \frac{3}{2}x^2$ is decreasing or increasing. (Algebraically)

$f'(x) = 3x^2 - 3x$

Dec: $3x^2 - 3x < 0$

$(3x)(x-1) < 0$

~~$3x < 0$ and $x - 1 > 0$~~

~~$x < 0$ and $x > 1$~~

$3x > 0$ and $x - 1 < 0$

$\boxed{x > 0 \text{ and } x < 1}$

$0 < x \leq 1$

Inc: $3x^2 - 3x > 0$

$(3x)(x-1) > 0$

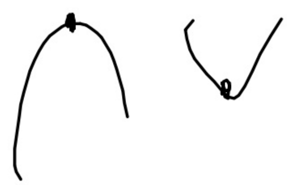
$3x > 0$ and $x - 1 > 0$ $\quad (1, \infty)$

$x > 0$ and $x \geq 1$

$3x < 0$ and $x - 1 < 0$ $\quad (-\infty, 0)$

$x < 0$ and ~~$x < 1$~~

0, 1 → C.V.



Example

VA: $x+2=0$

$x=-2$.

f $x^2(x+2)^{-1}$

Find the open intervals on which $y = \frac{x^2}{x+2}$ is decreasing or increasing.

$$y' = \frac{2x(x+2)-x^2}{(x+2)^2} = \frac{\boxed{x^2+4x}}{(x+2)^2}$$

$\boxed{(-4,-2)\cup(-2,0)}$ $\cancel{(-4,0)}$ $-4<x<0$

Dec.

CV: $x^2+4x=0$

$x(x+4)=0$

$x=0,-4$

relative max/min

"mountain or valley"

Inc

$x(x+4)>0$

$x>0$ and $(x+4)>0$

$x>0$ and $x>-4$

$x>0$ and $(x+4)<0$

$x<0$ and $(x+4)<0$

$\cancel{x<0 \text{ and } x<-4}$

$\boxed{(-\infty,-4)\cup(0,\infty)}$

$x(x+4)<0$

$x<0$ and $x+4>0$

$x<0$ and $x>-4$

$\cancel{x>0 \text{ and } x+4<0}$

$\cancel{x>0 \text{ and } x<-4}$

First Derivative Test

Putting all of this together, we come up with the **First Derivative Test**, which allows us to find all of the relative minimums and maximums.

The First Derivative Test- Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows.

dec, inc.

1. If $f'(x)$ changes from **negative to positive** at c, then f has a **relative minimum** at $(c, f(c))$. inc dec
2. If $f'(x)$ changes from **positive to negative** at c, then f has a **relative maximum** at $(c, f(c))$.
3. If $f'(x)$ either positive or negative on both sides of c, then $f(c)$ neither a relative minimum nor a relative maximum.

Example

Find the relative extrema of $f(x) = x^3 - \frac{3}{2}x^2$

$f'(x) = 3x^2 - 3x$

CV: $3x^2 - 3x = 0$.

$(3x)(x-1) = 0$

$x = 0, 1$ CV

-1	0	$\frac{1}{2}$	1	2
$+$		$-$		$+$

relative max @ $(0, 0)$

relative min @ $(1, -0.5)$

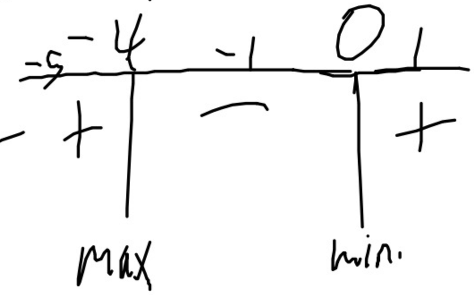
Example

Prev. ex.

$CV = 0, -4$

Find the relative extrema of $y = \frac{x^2}{x+2}$

$$y' = \frac{x^2+4x}{(x+2)^2} = \frac{x(x+4)}{(x+2)^2}$$



relative max @ $(-4, -8)$

relative min @ $(0, 0)$

Example

Find the relative extrema of the function $f(x) = \frac{1}{2}x - \sin x$

$5\pi/6$ $\frac{\sqrt{3}}{2}$

CV: $0 = \frac{1}{2} - \cos x$

$f'(x) = \frac{1}{2} - \cos x.$

$\frac{1}{2} = \cos x.$

$x = \pi/3, \; 5\pi/3$

relative min @ $\left(\pi/3, \; \pi/6 - \frac{\sqrt{3}}{2}\right)$

relative max @ $\left(5\pi/3, \; 5\pi/6 - \frac{\sqrt{3}}{2}\right)$

0	$\pi/3$	$5\pi/3$	2π
$-$	$+$	$-$	
	min	max	

Example

Find the relative extrema of $f(x) = (x^2 - 4)^{\frac{2}{3}}$

Example

Find the relative extrema of $f(x) = \frac{x^4+1}{x^2}$

Homework 10/26

3.3 #1-8, 9-15 (odd), 17-37 (e.o.o), 39-45 (odd)