Warm Up 9/29

1. Find the domain of the function

a. $f(x) = x^2 + 4x - 1$ b. $g(x) = \sqrt{9 - x^2}$ c. $f(x) = \frac{1}{x^2 - x}$

$(-\infty, \infty)$.

$9 - x^2 \geq 0$

$x^2 \leq 9$

$x \leq 3$

$x \geq -3$

$-3 \leq x \leq 3$

$[-3, 3]$.

$x^2 - x = 0$

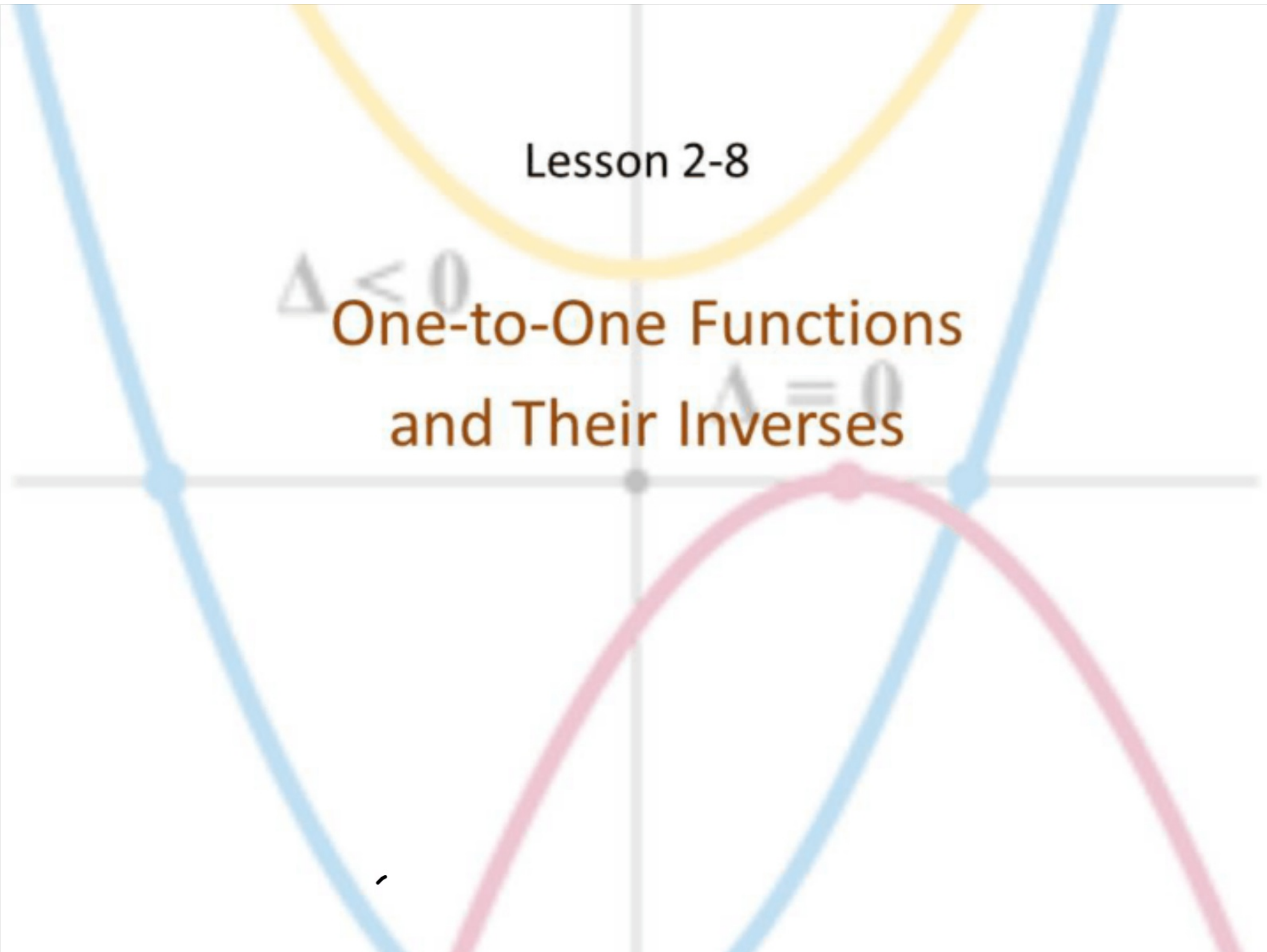
$x(x-1) = 0$

$x \neq 0, 1$

$(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

Lesson 2-8

One-to-One Functions and Their Inverses



Objective

Students will…

- Be able to define one-to-one functions.
- Be able to prove whether a given function is one-to-one, using horizontal line test and algebraically.
- Be able to find the inverse function of one-to-one functions.

For every output is there one input? NO

One-to-One Functions

$y = x^2$

$f(x) = 1$

$x = 1$ or

Function is defined as a relation having one output, per input. This only deals with the **number** of outputs, not necessarily the **type** of outputs. A one-to-one function is a function where no input shares a same output with another input. In other words,

$$f(x_1) = f(x_2) \text{ if and only if } x_1 = x_2$$

or

$$f(x_1) \neq f(x_2) \text{ if and only if } x_1 \neq x_2$$

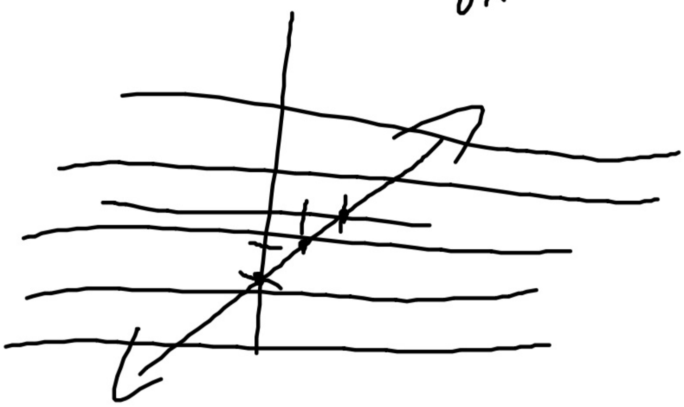
Again, the definition of a function only deals with the number of outputs. Two different inputs could share the same output, as long as they both have **one** single output.

Horizontal Line Test

One of the ways to visually identify one-to-one functions is using the horizontal line test, which simply states that a function is one-to-one if and only if no horizontal line intersects its graph more than once.

Ex. Use the horizontal line test to determine whether the following functions are one-to-one.

a. $f(x) = x - 2$ one to one



b. $f(x) = x^2$ Not one to

Proving One-to-one-"ness"

All even powered poly functions are not One to one

Sometimes horizontal line test may not be suitable, because drawing the graph of a function may be difficult. In this case, we need to determine whether a function is one-to-one algebraically. $f(x_1)=f(x_2) \iff x_1=x_2$

Ex. Show that the following function is one-to-one

1. $f(x) = 3x + 4$

$f(x_1)=f(x_2)$

$3x_1+\cancel{4}=3x_2+\cancel{4}$

$-4 \qquad -4$

$\frac{\cancel{3}x_1}{\cancel{3}}=\frac{\cancel{3}x_2}{\cancel{3}}$

$x_1=x_2$

1 - 1

2. $g(x) = x^2$

$g(x_1)=g(x_2)$

$\sqrt{x_1^2}=\sqrt{x_2^2}$

$\pm x_1=\pm x_2$

Not 1 - 1

3. $f(x) = x^3$

$f(x_1)=f(x_2)$

$\sqrt[3]{x_1^3}=\sqrt[3]{x_2^3}$

$x_1=x_2$

4. $h(x) = 4x^4$

Not 1 - 1.

Examples

$\cancel{x^3 - 2x^2 + 3}$

Algebraically, show whether the following functions are one-to-one

a. $f(x) = x + 2$ b. $g(x) = 6x^2 - 4$ c. $h(x) = 9x^3 - 7$

$f(x_1) = f(x_2)$

$x_1 + \cancel{2} = x_2 + \cancel{2}$

$-2 \quad -2$

$x_1 = x_2$ ✓

1-1

NOT

1-1

$h(x_1) = h(x_2)$

$\cancel{9}x_1^3 - \cancel{7} = \cancel{9}x_2^3 - \cancel{7}$

$+7 \quad +7$

$\div 9 \quad \div 9$

$\sqrt[3]{x_1^3} = \sqrt[3]{x_2^3}$

$x_1 = x_2$ ✓

Inverse Functions

The whole point of finding out whether a function is one-to-one or not has to do with inverse functions. For any one-to-one function, an inverse function must exist.

$y = x^2$

$4 \Rightarrow x = 2, -2$

Inverse functions is the "opposite" function. ~~By~~ definition, for a function f, let $f(x) = y$. Then, the inverse function f^{-1},

what is the inverse of x^2

$f^{-1}(y) = x$, for any y.

You can also think inverse function as the function that "undo's" the its original function.

Example

Assume f is one-to-one...

If $f(5) = 18$, find $f^{-1}(18) = 5$ If $f^{-1}(3) = 6$, find $f(6) = 3$

If $f(x) = 3x - 2$, find $f^{-1}(\overset{in}{16}) = \boxed{6}$

$16 = 3x - 2$

$18 = 3x$

$6 = x$

$\cancel{f(16) = 3(16) - 2}$

How to find the inverse function

1. Write "y =" instead of "f(x) ="

2. ~~Replace the~~ switch the "y" and the "x"

3. Solve the equation for "y"

4. The resulting equation is the inverse function, $f^{-1}(x)$

Example

1. $f(x) = x^3$

① 1 - 1 ?

$f(x_1) = f(x_2)$

$\sqrt[3]{x_1^3} = \sqrt[3]{x_2^3}$

$x_1 = x_2$

② $y = x^3$

$\sqrt[3]{y} \quad x = \sqrt[3]{y^3}$

$y = \sqrt[3]{x}$

$\boxed{f^{-1}(x) = \sqrt[3]{x}}$

2. $f(x) = x + 1$

① 1 - 1 ?

$f(x_1) = f(x_2)$

$x_1 + 1 = x_2 + 1$
$\quad -1 \qquad\quad -1$

$x_1 = x_2$

② $y = x + 1$

$x = y + 1$
$-1 \qquad -1$

$y = x - 1$

$\boxed{f^{-1}(x) = x - 1}$

3. $f(x) = 2x - 3$

① 1 - 1 ?

$f(x_1) = f(x_2)$

$2x_1 - \cancel{3} = 2x_2 - \cancel{3}$
$\quad +3 \qquad\qquad +3$

$\frac{\cancel{2}x_1}{\cancel{2}} = \frac{\cancel{2}x_2}{\cancel{2}} \Rightarrow x_1 = x_2$

② $y = 2x - 3$

$x = 2y - 3$
$+3 \qquad +3$

$\frac{x+3}{2} = \frac{2y}{2} \Rightarrow y = \frac{x+3}{2}$

$\boxed{f^{-1}(x) = \frac{x+3}{2}}$

Example

Remember! Only one-to-one functions can have an inverse function. So, it's important to make sure that the function is one-to-one before you try to find its inverse.

I.E. $f(x) = x^2$ does not have an inverse function!

Homework 9/29

TB pg. 230-231

#1-13 (e.o.o), 17, 19, 33, 37, 45