Lesson 3-2

Rolle's Theorem
and the
Mean Value Theorem

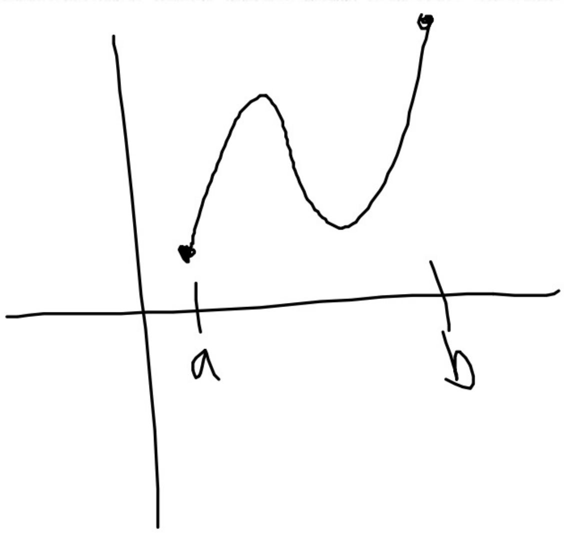
Objective

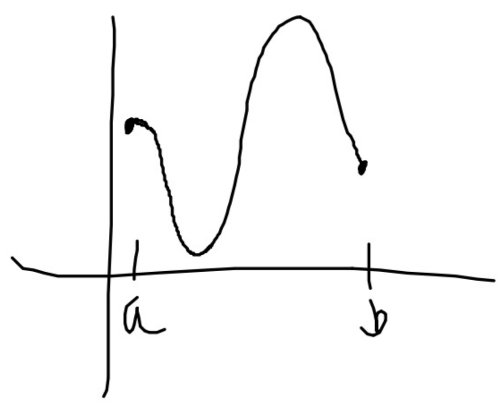
Students will…

- Be able to understand and use Rolle's Theorem.
- Be able to understand and use the Mean Value Theorem.

Rolle's Theorem

From the previous section, recall that the Extreme Value Theorem stated that continuous function on a closed interval must have both a minimum and a maximum within that interval. However, the theorem did not state where in the interval those extrema are located.

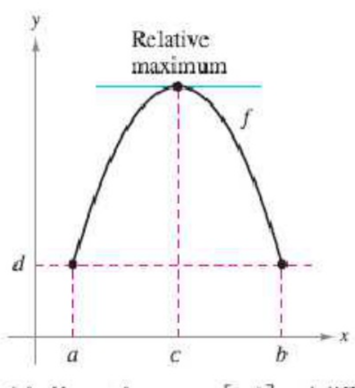




Rolle's Theorem

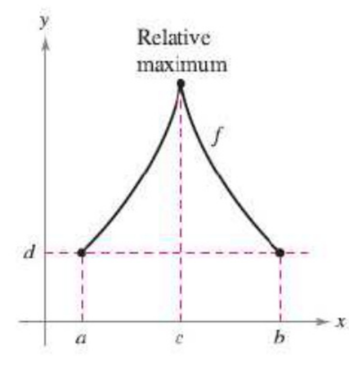
Rolle's Theorem, named after Michel Rolle, provides conditions that guarantee the existence of an extreme value in the **interior** of a closed interval (i.e. not an end point).

Rolle's Theorem- Let f be continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b). If $f(a) = f(b)$, then there is at least one number c in (a,b) such that $f'(c) = 0$.



(a) f is continuous on $[a, b]$ and differentiable on (a, b).

VS



(b) f is continuous on $[a, b]$.

Example

[-2, 2]

Let $f(x) = x^4 - 2x^2$. Find all of c in the interval $(-2, 2)$ such that $f'(c) = 0$.

$f'(x) = 4x^3 - 4x$

CV: $0 = 4x^3 - 4x$

$0 = 4x(x^2 - 1)$

$= 4x(x+1)(x-1)$

$x = 0, -1, 1$

Mean Value Theorem

The biggest application of the Rolle's Theorem, however, is proving one of the most important theorems in all of Calculus.

The Mean Value Theorem- If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$ = avg. rate of change

Instantaneous ROC ↗

The keyword in understanding this theorem is the word **continuous**!

Think, if you travelled at an average speed of 75mph ($\frac{f(b)-f(a)}{b-a}$), then you must have at some point reached that speed ($f'(c)$).

Example

$4x^{-1}$

Given $f(x) = 5 - \left(\frac{4}{x}\right)$ on an interval $(1, 4)$ does $f'(c) = \frac{f(4)-f(1)}{4-1}$? $= \frac{4-1}{3} = \frac{3}{3} = 1$

$f'(x) = 0 + 4x^{-2}$

$1 = \frac{4}{x^2}$

$x = 2 \in (1, 4)$

contained in

Example

Two stationary patrol cars equipped with radar are 5 miles apart on a highway. As a truck passes the first patrol car, its speed is clocked at 55 mph. Four minutes later, when the truck passes the second patrol car, its speed is clocked at 50 mph. Prove that the truck must have exceeded the speed limit of 55 mph at some time during the 4 minutes.

$t = 0, 4$

$f(4) = 50$

$f(0) = 55$

$\frac{50-55}{4-0} = \frac{-5}{4} = 1\frac{1}{4} = 1.25$ m/min. $\times 60 = 75$ mph. $0 \le c \le 4$

By MVT, there must be $t = c$, such that $f'(c) = 75 > 55$ mph.

Homework 10/25

3.2 #1-8, 11-23 (odd), 29, 39-45 (odd), 51, 57