Lesson 4-1b

# Antiderivatives

# And

# Indefinite Integration II

# Objective

Students will…

- Be able to use integral notation.
- Be able to apply some of the basic integration rules.

$\frac{d}{dx}$ $\frac{d}{dt}$

# Integral Notation

$\int_a^b$ definite $\int$ indef.

In the modern world, the notation for antiderivatives is the **integral notation**, namely,

$$\int f(x)\, \boxed{dx} = F(x) \boxed{+ C}$$

This would be read as *the indefinite integral of f with respect to x*. So the $dx$ serves to identify $x$ as the variable of integration.

# Example

Evaluate the following indefinite integrals.

a. $\int 3x\, dx$

$= \frac{3}{2}x^2 + C$

b. $\int \frac{1}{x^3} = \int x^{-3}$

$= \frac{1}{-2}x^{-2} + C$

c. $\int \sqrt{x} = \int 1x^{1/2}$

$= \frac{2}{3}x^{3/2} + C.$

$\frac{1}{\frac{3}{2}}$

# Example

d. $\int \boxed{2} \sin x \; dx$

$2\int \sin x \, dx$

$2(-\cos x + C)$

$= \boxed{-2\cos x + C}$

e. $\int \frac{\sin x}{\cos^2 x} \, dx$

$= \int \frac{\sin x}{\cos x} \frac{1}{\cos x} \, dx$

$= \int \tan x \sec x \, dx.$

$= \boxed{\sec x + C}$

# Example

f. $\int \frac{\cos x \sin^2 x + \cos^3 x}{\cos^3 x}\, dx$

$$= \int \frac{\cancel{\cos x} \sin^2 x}{\cos^2 x} + \frac{\cancel{\cos^3 x}}{\cancel{\cos^3 x}}\, dx. = \int \overbrace{\tan^2 x + 1}^{\sec^2 x.}\, dx.$$

$$= \int \sec^2 x\, dx.$$

$$= \boxed{\tan x + C}$$

$$\int \tan x\, dt.$$

# Example

g. $\int \frac{x+1}{\sqrt{x}}\,dx = \int (x^{-1/2})(x^1+1)\,dx. = \int x^{1/2} + x^{-1/2}\,dx.$

$$= \int x^{1/2}\,dx + \int x^{-1/2}\,dx.$$

$$= \frac{2}{3}x^{3/2} + 2x^{1/2} + C$$

$$= \boxed{\frac{2}{3}x^{3/2} + 2x^{1/2} + C}$$

$s(t)$ $s'(t)$

# Example

$f'$ – 1st derivative

$t=0$. $t$

A ball is thrown upward with an initial velocity of 64 ft/s from an initial height of 80 ft.

$f$ $f''$ → acceleration of gravity $=-32$ ft/sec/sec.

a. Find the position function of height $s$ as a function of the time $t$.

$s(0)=80$

$s'(0)=64$

$s''(t)=-32$

$\int -32\,dt=-32t+C$

$64=-32(0)+C$

$64=C$

$\int -32t+64\,dt$

$=-16t^2+64t+C$

$80=-16(0)^2+64(0)+C$

$\boxed{s(t)=-16t^2+64t+80}$

# Example

b. When does the ball hit the ground?

$$0 = -16t^2 + 64t + 80$$

$$0 = \cancel{-16}(\underbrace{t^2 - 4t - 5})$$

$$0 = (t-5)(t+1)$$

$$t = 5$$

Homework 12/12

4.1- #5-8, 17-41 (e.o.o), 55, 59, 62, 63, 70, 72, 73