Lesson 3-6

Curve Sketching

Objective

Students will…

- Be able analyze and sketch the graph of a function.

Guidelines for Curve Sketching

GUIDELINES FOR ANALYZING THE GRAPH OF A FUNCTION

1. Determine the domain and range of the function.
2. Determine the intercepts, asymptotes, and symmetry of the graph.
3. Locate the x-values for which $f'(x)$ and $f''(x)$ either are zero or do not exist. Use the results to determine relative extrema and points of inflection.

4. Label as much info as you can.

Example

Sketch the graph of $f(x) = \frac{2(x^2-9)}{x^2-4}$

$-2 \quad 0 \quad 2$: $- \mid - \mid + \mid +$ f

$-2 \quad 2$: $- \mid + \mid -$ g

$f'(x) = \frac{2(2x)(x^2-4) - 2x(2x^2-18)}{(x^2-4)^2}$

$= \frac{\cancel{4x^3} - 16x - \cancel{4x^3} + 36x}{(x^2-4)^2}$

$= \boxed{\frac{20x}{(x^2-4)^2}}$

$f''(x) = \frac{20(x^2-4)^2 - 2(x^2-4) \cdot 2x \cdot 20x}{(x^2-4)^4}$

$= \frac{20\cancel{(x^2-4)}(-3x^2-4)}{(x^2-4)^3}$

X-int: $x = \pm 3$

y-int: $y = \frac{9}{2}$

V-asym: $x = \pm 2$

h-asym: $y = 2$

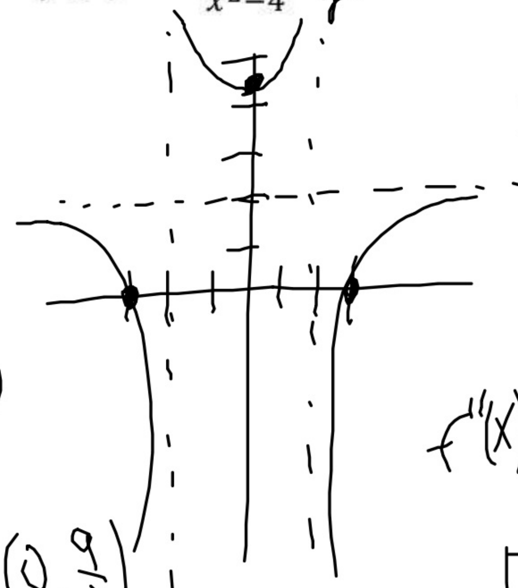
CV: $x = 0, \pm 2$

Inc/Dec: Dec: $(-\infty, 0)$ Inc: $(0, \infty)$

Rel ext: Rel min @ $(0, \frac{9}{2})$

CD: $(-\infty, -2) \cup (2, \infty)$

CU: $(-2, 2)$ pts. inf: $(\ldots)$



max 0   2   4 min

+ | − | − | +

Example

Sketch the graph of $f(x) = \frac{x^2-2x+4}{x-2}$

$f' \quad 16 - 8 + 4$

$g \quad 2$

x-int: $x^2 - 2x + 4 = 0$

$x = \frac{2 \pm \sqrt{4 - 4(1)(4)}}{2}$ = no sol.

y-int: $(0, -2)$

VA: $x - 2 = 0 \Rightarrow x = 2$

HA: DNE

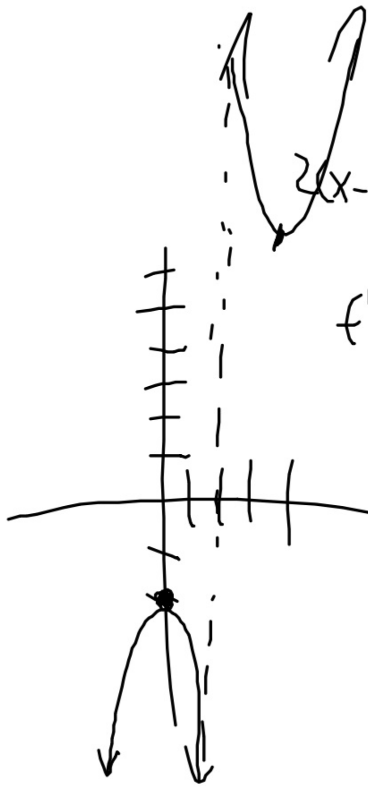
CV: $x = 0, 4, 2$

Inc: $(-\infty, 0) \cup (4, \infty)$

Dec: $(0, 2) \cup (2, 4)$

Rel min: $(4, 6)$

Rel max: $(0, -2)$



$$f'(x) = \frac{(2x-2)(x-2) - (x^2-2x+4)}{(x-2)^2}$$

$$= \frac{2x^2 - 4x \cancel{-2x+4} - x^2 \cancel{+2x-4}}{(x-2)^2}$$

$$= \frac{x^2-4x}{(x-2)^2} = \frac{x(x-4)}{(x-2)^2} = 0$$

$2(x-2)$

$$f''(x) = \frac{(2x-4)(x-2)^2 - 2(x-2)(x^2-4x)}{(x-2)^4}$$

$$= \frac{2(x-2)^3 - 2(x-2)(x^2-4x)}{(x-2)^4}$$

Example

Sketch the graph of $f(x) = \frac{1x}{\sqrt{x^2+2}} = x(x^2+2)^{-1/2}$

$0 \leftarrow$ pt of inf.

$+ \mid -$

CU $\mid$ CD

$x^2+2=0$

x-int: $x=0$ $(0,0)$

y-int: $(0,0)$

VA: DNE

HA: $y=1$

CV: DNE

$(f'>0)$

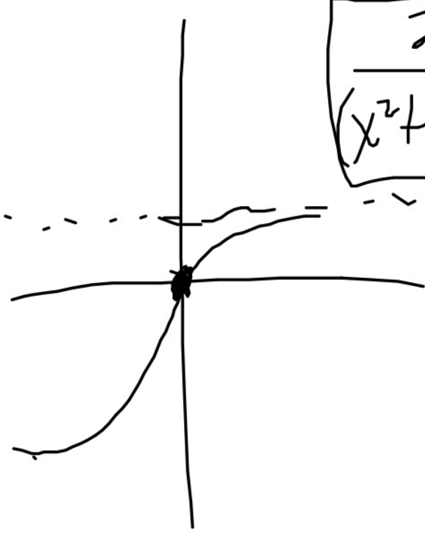
No relative extrema

CV: $x=0$

CU: $(-\infty,0)$

CD: $(0,\infty)$

POI: $(0,0)$



$f'(x) = (x^2+2)^{-1/2} + -\frac{1}{2}(x^2+2)^{-3/2} \cdot 2x \cdot x$

$= (x^2+2)^{-1/2} - x^2(x^2+2)^{-3/2}$

$= \frac{(x^2+2)^{2/2}}{(x^2+2)^{2/2}}\frac{1}{(x^2+2)^{1/2}} - \frac{x^2}{(x^2+2)^{3/2}}$

$\boxed{\frac{2}{(x^2+2)^{3/2}}} = \frac{2}{(x^2+2)(\sqrt{x^2+2})}$

$f''(x) = \frac{0 - 4x\frac{3}{2}(x^2+2)^{1/2}}{(x^2+2)^3}$

$= \frac{-6x(x^2+2)^{1/2}}{(x^2+2)^3}$ $+$

Example

Sketch the graph of $f(x) = x^4 - 12x^3 + 48x^2 - 64x$

$f''(x) = 12x^2 - 72x + 96$

$= 12(x^2 - 6x + 8) = \boxed{12(x-4)(x-2)}$

1 −12 +48 −64

X-int: (0,0), (4,0)

$= x(x^3 - 12x^2 + 48x - 64)$

64 − 192 + 192 − 64

$$\begin{array}{c|cccc} 4 & 1 & -12 & 48 & -64 \\ + & \downarrow & 4 & -32 & 64 \\ \hline & 1 & -8 & 16 & 0 \end{array}$$

$= x(x-4)^3$

y-int: (0,0)

CV: x = 1, 4

Dec: $(-\infty, 1)$

Inc: $(1, \infty)$

Rel min: (1, −27)

f'' CV: x = 4, 2

CU: $(-\infty, 2) \cup (4, \infty)$

CD: (2, 4)

POI: (2,

$f'(x) = 4x^3 - 36x^2 + 96x - 64$

$0 = 4(x^3 - 9x^2 + 24x - 16)$

~~1 −9 +24 −16~~

$$\begin{array}{c|cccc} 1 & 1 & -9 & 24 & -16 \\ + & \downarrow & 1 & -8 & 16 \\ \hline & 1 & -8 & 16 & \boxed{0} \end{array}$$

$(x-1)(x^2 - 8x + 16)$

$f'(x) = (x-1)(x-4)^2$

1	4

− | + | +

2	4

+ | − | +

Example

Sketch the graph of $f(x) = \frac{\cos x}{1+\sin x}$

$\frac{3\pi}{2}$: $-\,|\,-$

$\frac{\pi}{2}$, $\frac{3\pi}{2}$: $+\,|\,-\,|\,+$

x-int: $x = \frac{\pi}{2}, \cancel{\frac{3\pi}{2}}$

$\left(\frac{\pi}{2}, 0\right), \cancel{\left(\frac{3\pi}{2}, 0\right)}$

y-int: $(0, 1)$

VA: $1+\sin x = 0$

$\sin x = -1$

$x = \frac{3\pi}{2}$

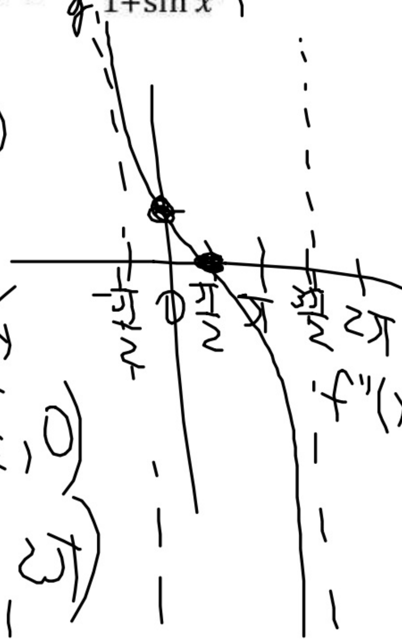
CV: $x = \frac{3\pi}{2}$

Dec: $\left(0, \frac{3\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$

POI: $\left(\frac{\pi}{2}, 0\right)$

f'' CV: $x = \frac{\pi}{2}, \frac{3\pi}{2}$

CU: $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$ CD: $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$



$$f'(x) = \frac{-\sin x(1+\sin x) - \cos x(\cos x)}{(1+\sin x)^2}$$

$$= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2}$$

$$= \frac{-(\sin x + \cancel{\sin^2 x + \cos^2 x})}{(1+\sin x)^2}$$

$$= \frac{-\cancel{(1+\sin x)}}{(1+\sin x)^{\cancel{2}}} = \boxed{\frac{-1}{1+\sin x}}$$

$$f''(x) = \frac{0 - \cos x(-1)}{(1+\sin x)^2} = \boxed{\frac{\cos x}{(1+\sin x)^2}}$$

Homework 11/2

3.6 #1-4, 7-31 (e.o.o), 39-45 (e.o.o)