$a+bi$.
real imag.

Warm Up 10/29

$\frac{1}{2} < \frac{1}{0.5}$

$1/2 < 2$

Evaluate the expression

1. $(x-(\overset{-2+i}{i-2}))(x-(\overset{2+i}{i+2}))$ $= (x-i+2)(x-i-2)$

$= x^2 - ix \cancel{-2x} - ix + \overset{-1}{\cancel{2}} \cancel{+2i} \cancel{+2x} \cancel{-2i} - 4.$

$= x^2 - 2ix - 5$

True or false?

3. $\frac{1}{87} > \frac{1}{86}$

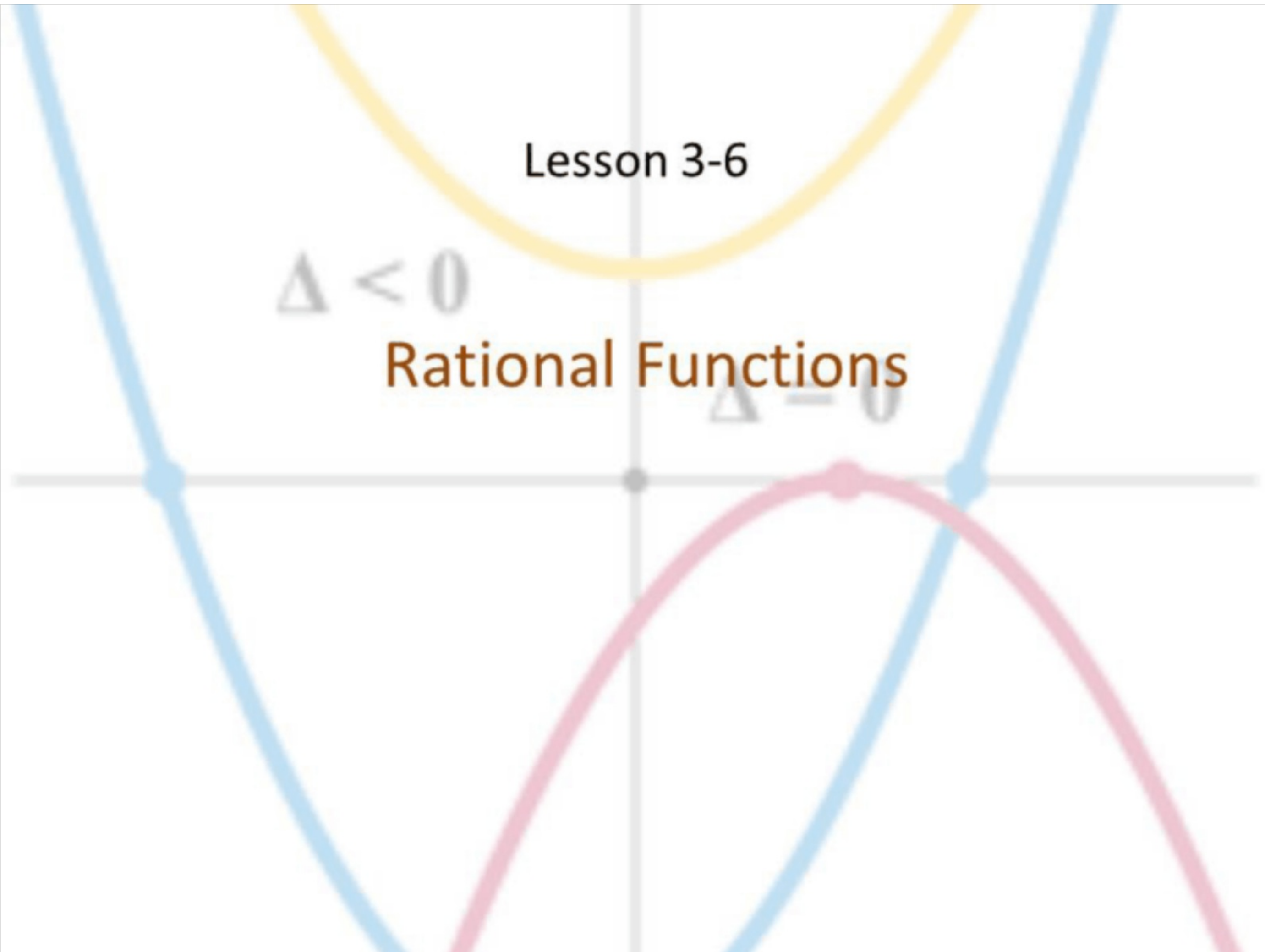
F

4. $\frac{x+3}{x+1} > \frac{x+2}{x+1}$

T

Lesson 3-6

Rational Functions



Objective

Students will…

- Be able to understand what rational functions are and their behaviors.
- Be able to find the x and the y intercepts of rational functions.

Rational Functions

Whenever we hear the word "rational" in mathematics, it'd be safe to say many of us think of fractions. Hence, a rational function would be most commonly described as a "fractional" function. This is in essence true!

A rational function is a function of the form

$$r(x) = \frac{P(x)}{Q(x)}$$

$Q(x) \neq 0$ $\quad \frac{3}{2} = \frac{0x^2+0x+3}{0x^6+2}$

where P and Q are polynomials. We are also assuming that $P(x)$ and $Q(x)$ have no factor in common, i.e. they are completely reduced.

Behaviors of Rational Functions

Rational functions are often given special attention because, while they fit the standard definition of a function (one output for every input), they are quite unique in terms of their behaviors and structure. Consider the following rational function,

$$f(x) = \frac{1}{x}$$

We can already see that there is something we need to make sure of, and that is the fact that $x \neq 0$, since a fraction is not defined when the denominator is a zero.

Behaviors of Rational Functions

Also, as x or the denominator **<u>increases</u>**, the overall function **<u>decreases</u>**, and as x or the denominator **<u>decreases</u>**, the overall function **<u>increases</u>**.

$\frac{1}{x}$

Ex. $\frac{1}{2} > \frac{1}{12} > \frac{1}{45667}$

So, the behavior of this rational function, $f(x) = \frac{1}{x}$ can be written as,

$\boxed{\lim_{x\to\infty}} f(x) = 0$ and $\boxed{\lim_{x\to 0}} f(x) = \infty$

"The limit of $f(x)$ as x approaches infinity is 0"

"The limit of $f(x)$ as x approaches 0 is infinity."

X and the Y-Intercepts of Rational Functions

Although we have observed how rational functions behave in a unique way, the concept of finding the x and the y intercepts remain the same for all functions.

$x-2=0$

Ex. Find the x and the y-intercepts of the function $f(x) = \frac{x-2}{3}$

Y-int:

$f(0) = \frac{0-2}{3}$

$= -2/3$

$(0, -2/3)$

X-int:

$(3)0 = \frac{x-2}{\cancel{3}}(\cancel{3})$

$0 = x-2$

$2 = x$

Examples

Find the x and the y intercepts of the following rational functions

1. $f(x) = \frac{1}{x}$

y-int: $\cancel{f(0) = \frac{1}{0}}$

DNE

x-int: $\frac{1}{x} \neq 0$

DNE.

2. $r(x) = \frac{x}{2}$

y-int: $f(0) = \frac{0}{2}$

$(0, 0)$

x-int: $\frac{x}{2} = 0$

$x = 0$

$(0, 0)$

3. $g(x) = \frac{x-5}{x-2}$

y-int: $g(0) = \frac{0-5}{0-2}$

$= 5/2$

$(0, 5/2)$

x-int: $x - 5 = 0$

$x = 5$

$(5, 0)$

Examples

Find the x and the y intercepts of the following rational functions

1. $f(x) = \frac{x^2-3x-18}{x+4}$ $\left(0, -\frac{9}{2}\right)$

y-int: $\frac{-18}{4} = -\frac{9}{2}$

x-int: $x^2-3x-18=0$

$(x-6)(x+3)=0$

$x=6,-3$

$(6,0),(-3,0)$

2. $r(x) = \frac{x^2+6}{2}$

y-int: $r(0) = \frac{0+6}{2} = (0,3)$

x-int: $x^2+6=0$ DNE

$\cancel{x^2=-6}$

$\cancel{x=\pm i\sqrt{6}}$

$\cancel{(i\sqrt{6},0),(-i\sqrt{6},0)}$

Homework 10/29

TB pg. 313 #5-14 (Just find the intercepts)