Warm Up 10/29

Evaluate the expression

1. $\cancel{(x-(i-2))(x-(i+2))}$

True or false?

3. $\frac{1}{87} > \frac{1}{86}$

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4. $\frac{x+3}{x+1} > \frac{x+2}{x+1}$

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Lesson 3-6

Rational Functions

Objective

Students will…

- Be able to understand what rational functions are and their behaviors.
- Be able to find the x and the y intercepts of rational functions.

Rational Functions

Whenever we hear the word "rational" in mathematics, it'd be safe to say many of us think of fractions. Hence, a rational function would be most commonly described as a "fractional" function. This is in essence true!

A rational function is a function of the form

$$r(x) = \frac{P(x)}{Q(x)}$$

ex. $\frac{3}{5} = \frac{3x^2}{5x^2} = \frac{3x^0}{5x^0}$

where P and Q are polynomials. We are also assuming that $P(x)$ and $Q(x)$ have no factor in common, i.e. they are completely reduced.

Behaviors of Rational Functions

Rational functions are often given special attention because, while they fit the standard definition of a function (one output for every input), they are quite unique in terms of their behaviors and structure. Consider the following rational function,

$x \neq 0$

$$f(x) = \frac{1}{x}$$

We can already see that there is something we need to make sure of, and that is the fact that $x \neq 0$, since a fraction is not defined when the denominator is a zero.

Behaviors of Rational Functions

$\frac{1}{x}$

Also, as x or the denominator **increases**, the overall function **decreases**, and as x or the denominator **decreases**, the overall function **increases**.

$\frac{1}{0.0001} > \frac{1}{0.1} > \frac{1}{0.9} >$ Ex. $\frac{1}{1} > \frac{1}{2} > \frac{1}{12} > \frac{1}{45667} > \frac{1}{50000} > \frac{1}{3.1 \times 10^{23}}$

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So, the behavior of this rational function, $f(x) = \frac{1}{x}$ can be written as,

$$\lim_{x \to \infty} f(x) = 0 \qquad \text{and} \qquad \lim_{x \to 0} f(x) = \infty$$

convergent Divergent

"The limit of $f(x)$ as x approaches infinity is 0"

"The limit of $f(x)$ as x approaches 0 is infinity."

X and the Y-Intercepts of Rational Functions

Although we have observed how rational functions behave in a unique way, the concept of finding the x and the y intercepts remain the same for all functions.

Ex. Find the x and the y-intercepts of the function $f(x) = \frac{x-2}{3}$

$x-2=0$

$x=2$

Y-int: $f(0) = \frac{0-2}{3} = -2/3$

$(0, -2/3)$

X-int: $(3)0 = \frac{x-2}{3}(3)$

$0 = x-2$

$2 = x$

$(2, 0)$

Examples

Find the x and the y intercepts of the following rational functions

1. $f(x) = \frac{1}{x}$

y-int: $f(0) = \frac{1}{0} =$ und.

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X-int: $0 = \frac{1}{x}$

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2. $r(x) = \frac{x}{2}$

y-int: $f(0) = 0$

$(0, 0)$

X-int: $x = 0$

$(0, 0)$

3. $g(x) = \frac{x-5}{x-2}$

y-int: $g(0) = 5/2$

$(0, 5/2)$

X-int: $x - 5 = 0$

$x = 5$

$(5, 0)$.

Examples

Find the x and the y intercepts of the following rational functions

1. $f(x) = \frac{x^2-3x-18}{x+4}$

y-int: $\frac{-18}{4} = \frac{-9}{2}$

$(0, -9/2)$

x-int: $x^2-3x-18=0$

$(x-6)(x+3)=0$

$x=6,-3$

$\boxed{(6,0)\ (-3,0)}$

2. $r(x) = \frac{x^2+6}{2}$

y-int: $(0,3)$

x-int: $x^2+6=0$

$\sqrt{x^2}=\sqrt{-6}$

$x=\pm i\sqrt{6}$

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Homework 10/29

TB pg. 313 #5-14 (Just find the intercepts)