

Warm Up 11/12

Find x.

1) $2^x = 8$

$x = 3$

2) $4^x = 16$

$x = 2$

3) $3^x = 27$

$x = 3$

4) $5^x = 625$

$x = 4$

5) $e^x = 1$

$x = 0$

6) $10^x = 100000$

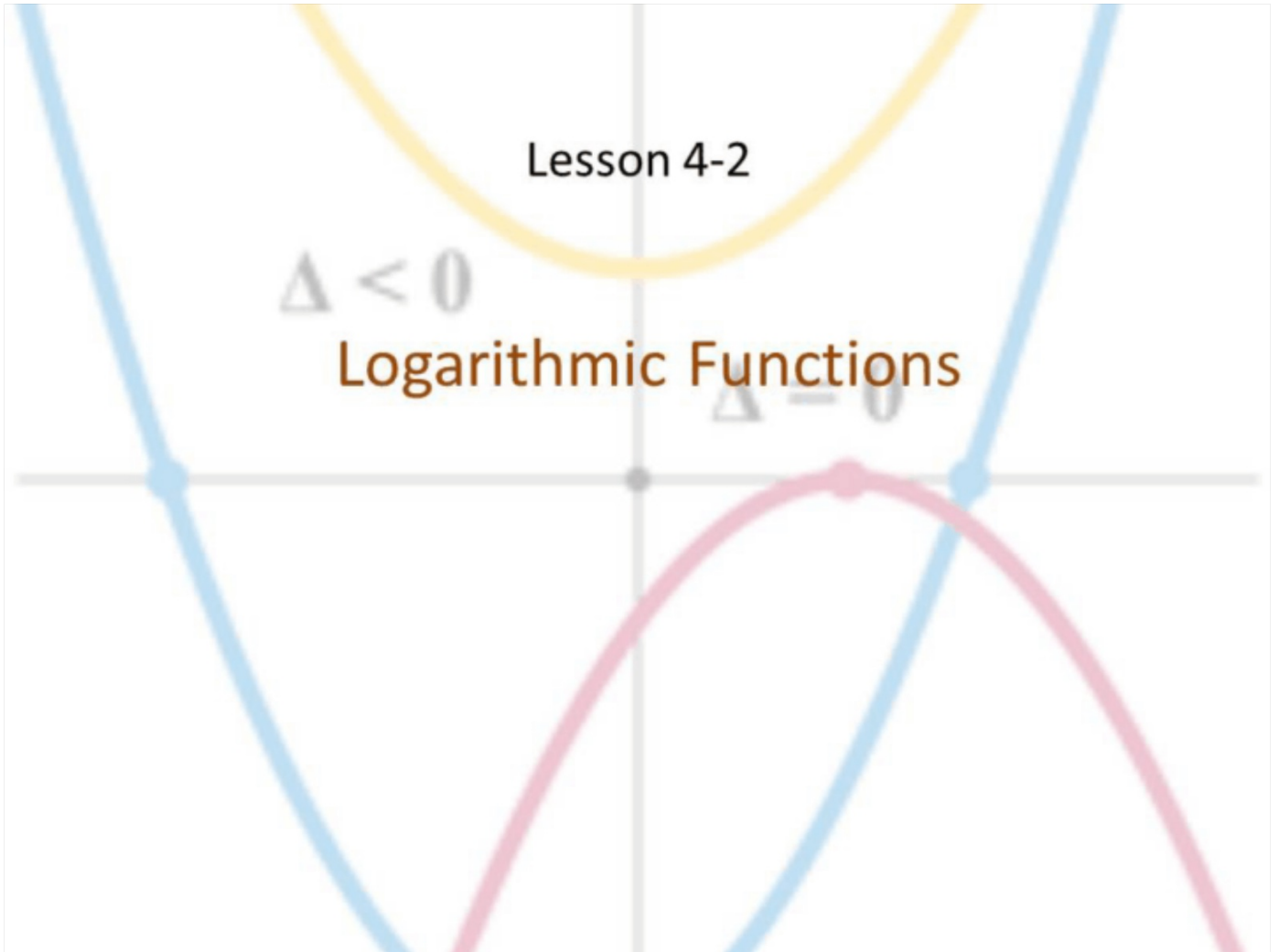
$x = 5$

Lesson 4-2

$\Delta < 0$

Logarithmic Functions

$\Delta = 0$



Objective

Students will...

- Be able to define the logarithmic function, as the inverse of the exponential function
- Be able to know and apply the properties of logarithms
- Be able to use calculators to compute logarithms

Inverses

$$\begin{array}{r} 2x + 3 = 4 \\ -3 \quad -3 \\ \hline 2x = 1 \\ \frac{2x}{2} = \frac{1}{2} \end{array}$$

In mathematics, **inverses** can be defined as the reverse operation (operations that revert the original operation).

We've dealt with plenty of examples of inverses in the past. Consider the following examples.

Starting with a ...

Multiplication and division: $2 \times a \div 2 = a$

Addition and subtraction: $2 + a - 2 = a$

Powers and roots: $\sqrt[2]{a^2} = a$

One-to-one Function and Inverse Function: $f^{-1}(a) = a$

$$\begin{array}{r} x = 6 \\ \frac{2x}{2} = \frac{12}{2} \\ \hline x = 3 \end{array}$$

$$f^{-1}(a) = a$$

The Inverse of Exponential Function 2^x .

With regards to the exponential function, there exists an inverse function which is called the **Logarithmic Function**.

Definition: Let a be a number with $a \neq 1$. The **logarithmic function** with base a , denoted by \log_a , is defined by

$$\log_a x = y \quad \text{if and only if} \quad a^y = x$$

So, here y is the **exponent** to which the base a must be raised to give x .

Examples

Revisiting our warm-up problems then...

1) $2^x = 8$

$x = 3$ means

$$\log_2 8 = 3$$

2) $4^x = 16$

$x = 2$ means

$$\log_4 16 = 2$$

3) $3^x = 27$

$x = 3$ means

$$\log_3 27 = 3$$

4) $5^x = 625$

$x = 4$ means

$$\log_5 625 = 4$$

5) $e^x = 1$

$x = 0$ means

$$\log_e 1 = 0$$

6) $10^x = 100000$

$x = 5$ means

$$\log_{10} 100000 = 5.$$

Properties of Logarithms

$$\log_2 8 = 3$$
$$\log_2 2^3 = 3$$

We are familiar with some of the properties of exponents. Here, we've established that \log_a is an exponent. Therefore, the properties of logarithms exist, much similar to the properties of exponents.

Property

Reason

1. $\log_a 1 = 0$

Anything raised to the zero power is 1

2. $\log_a a^1 = 1$

Anything raised to the 1st power is itself

3. $\log_a a^x = x$

a raised to the x power is a^x

4. $a^{\log_a x} = x$

$$\frac{2^x}{2} = x$$

$\log_a x$ is the power to which a must be raised to get x

$$a^{\log_a x} = x$$
$$a^x = \log_a x = a^x$$

Examples

For base 5...

By property 1:

$$\log_5 1 = 0$$

By property 2:

$$\log_5 5 = 1$$

By property 3:

~~$$\log_5 5^8 = 8$$~~

By property 4:

~~$$5^{\log_5 12} = 12$$~~

You try

For base 10...

By property 1:

$$\log_{10} 1 = 0$$

By property 2:

$$\log_{10} 10 = 1$$

By property 3:

$$\log_{10} 10^4 = 4$$

By property 4:

$$10^{\log_{10} 11} = 11$$

Common Logarithms

$$\begin{array}{l} 3^2 \\ 3^1 \end{array} \quad | + 1 = 2^1$$

2x (x¹)

In logarithms, base 10 is considered the "standard base." Therefore, it has a special name within the logarithmic function.

$$a_1 x^n$$

Common Logarithm- Base 10 said to be the **common base**, so any *log* base 10 is denoted without the base written:

$$\log_{10} 9 = \log 9$$

So always assume that *log* has base 10 if there is no base written.

$$\text{Ex. } \log 100 = 2 \quad \text{and} \quad \log 10 = 1$$

In Closing

Write the answer to the following questions and share with a partner.

1. What is $\log_9 81$ equal to, and why?

~~2. Explain why $a^{\log_a x} = x$~~

Homework 11/12

TB pg. 349-350 #1, 3-5, 9, 12, 15, 19, 29