Warm Up 10/27

Evaluate the expression

1. $\frac{2+\sqrt{-8}}{1+\sqrt{-2}} = \frac{2+2i\sqrt{2}}{1+i\sqrt{2}} \cdot \frac{(1-i\sqrt{2})}{1-i\sqrt{2}}$

$= \frac{2-2i^2\sqrt{4}}{1-i^2\sqrt{4}} = \frac{2+4}{1+2} = \frac{6}{3} = \boxed{2}$

2. $\frac{25}{4-3i} \cdot \frac{(4+3i)}{4+3i} = \frac{100+75i}{16-9i^2}$

$= \frac{100+75i}{25} = \boxed{4+3i}$

3. $2i(\frac{1}{2} - i)$

$= i - 2i^2$

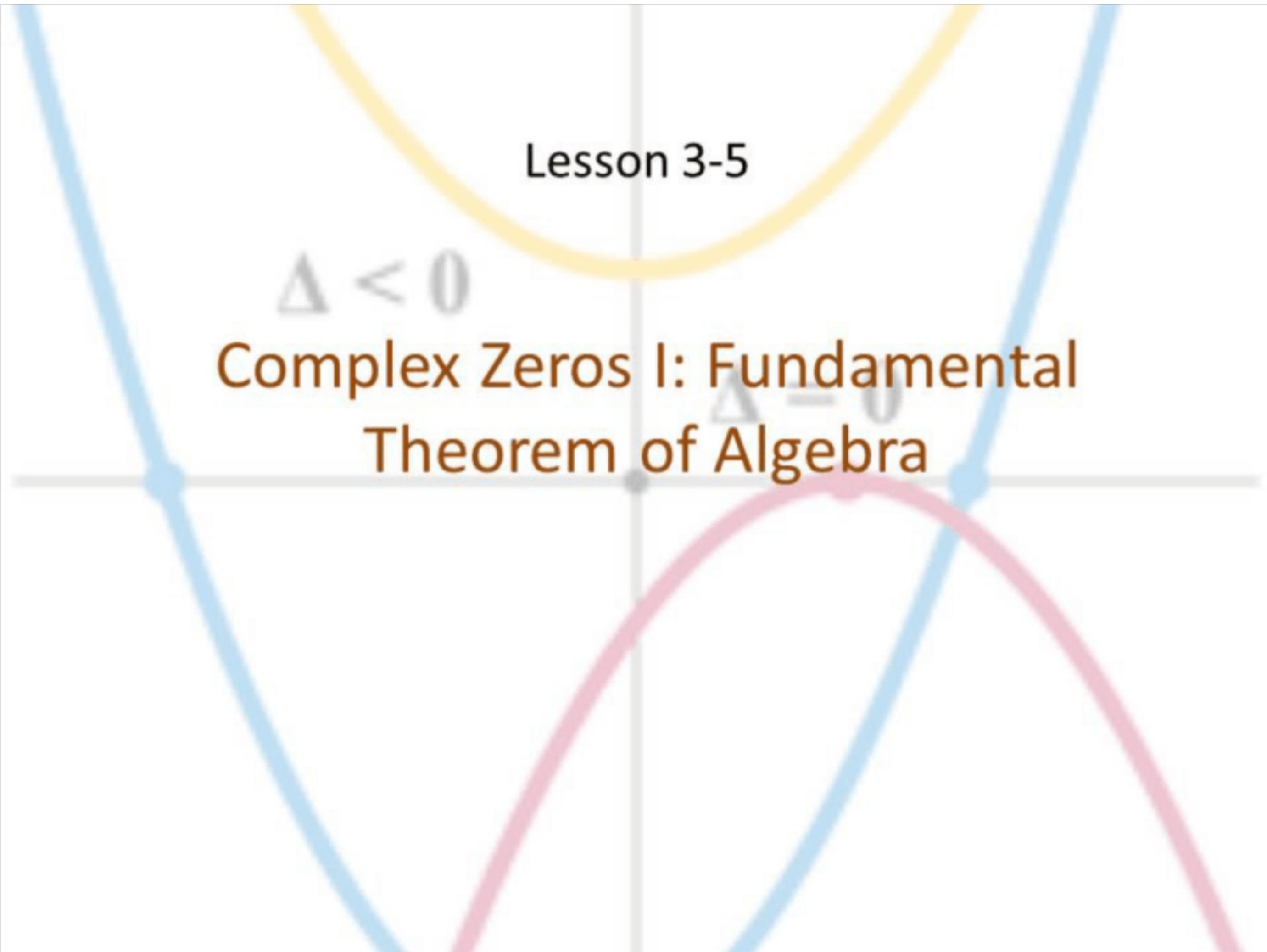
$= \boxed{i+2}$

4. $i^{1002} = (i^2)^{501}$

$= \boxed{-1}$

Lesson 3-5

Complex Zeros I: Fundamental Theorem of Algebra



Objective

Students will…

- Be able to understand what the Fundamental Theorem of Algebra says.
- Be able to factor any polynomial completely using a combination of factoring techniques, synthetic division, and quadratic formula.

Square Root of Negative Numbers

We observed in the past that real numbers alone had some limitations when solving for certain quadratic equations. This was due to the fact that some quadratics required taking the square root of a negative number. For example, to find the zeros of the following polynomial,

$$P(x) = x^2 - x + 1$$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)} \qquad \text{Quadratic Formula}$$

$$x = \frac{1 \pm \sqrt{1-4}}{2} = \frac{\mathbf{1 \pm \sqrt{-3}}}{\mathbf{2}}$$

In the past, we'd simply write, "no real solutions," for such equations. So, in order to solve all quadratic equations, mathematicians created an expanded number system called, *the Complex Number System*.

Fundamental Theorem of Algebra

18
9 2
3 3 $3^2 \cdot 2$

Now that we have eliminated the any limitations to solving quadratic equations by the use of complex numbers, the following theorems result:

$7 = 0i + 7$

Fundamental Theorem of Algebra- Every polynomial with complex coefficient has at least one complex zero.

Complete Factorization Theorem- Every polynomial can be factored completely into linear factors (i.e. $6x(x-1)^2(2x+5)^2 \ldots$)

Factoring a Polynomial Completely

We now use the various techniques that we have acquired to factor polynomials completely, and find all the zeros of any given polynomial.

Ex: Let $P(x) = (x^3 - 3x^2) + (x - 3)$. Find all the zeros and factor completely.

$$P(x) = x^2(x-3) + 1(x-3) = (x^2+1)(x-3)$$

$$= \boxed{(x-3)(x-i)(x+i)}$$

$$x^2+1=0$$

$$\sqrt{x^2} = \sqrt{-1}$$

$$x = \pm\sqrt{-1}$$

$$\boxed{\begin{array}{l} x = \pm i \\ x = 3 \end{array}}$$

$(x-4)$
$x=4$

Example

$x = \frac{2 \pm \sqrt{4-4(1)(2)}}{2} = \frac{2 \pm \sqrt{-4}}{2}$
$= \frac{2 \pm 2i}{2}$

Let $Q(x) = x^3 - 2x + 4$. Find all the zeros and factor completely.

$\frac{\text{Factors of } 4}{\text{Factors of } 1} = \pm \frac{1}{1}, \pm \frac{2}{1}, \pm \frac{4}{1} = \pm 1, \pm 2, \pm 4$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -2 & 4 \\ + & \downarrow & -2 & 4 & -4 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

$Q(x) = (x+2)(x^2-2x+2)$

$\boxed{x = -2, 1 \pm i}$

$\boxed{x = -2, 1+i, 1-i}$

$Q(x) = (x+2)(x-(1+i))(x-(1-i))$

$\boxed{= (x+2)(x-1-i)(x-1+i)}$

Zeros Theorem

We have another theorem regarding zeros or solutions of a polynomial.

<u>Zeros Theorem</u>- Every polynomial of degree $n \geq 1$ has **<u>exactly</u>** n zeros, provided that a zero of <u>multiplicity</u> k is counted k times.

deg 10

Example: $P(x) = (x-1)^3(x+2)^2(x+3)^5 = (x-1)(x-1)(x-1)(x+2)(x+2).$

Zeros: 1 -2 -3

Multiplicity: 3 + 2 + 5 = 10

So, here we see that $P(x)$ with degree 10 has exactly 10 zeros.

Example

16 4 4 8

Factor completely, find all zeros, and state the multiplicity of each zero.

$$P(x) = 3x^5 + 24x^3 + 48x$$

$$P(x) = 3x(x^4 + 8x^2 + 16) = 3x(x^2+4)(x^2+4)$$

$$= \boxed{3x(x-2i)^2(x+2i)^2} \quad \boxed{x=0}$$

$$x^2 + 4 = 0$$

$$\sqrt{x^2} = \sqrt{-4}$$

$$\boxed{x = \pm 2i}$$

Homework 10/27

TB pg. 298 #1-17 (e.o.o), 23