Warm Up 11/12

Find x.

$12^x = 4.4982$

1) $2^x = 8$

$x = 3$

2) $4^x = 16$

$x = 2$

3) $3^x = 27$

$x = 3$

4) $5^x = 625$

$x = 4$

5) $e^x = 1$

$x = 0$

6) $10^x = 100000$

$x = 5$

Lesson 4-2

Logarithmic Functions

Objective

Students will…

- Be able to define the logarithmic function, as the inverse of the exponential function
- Be able to know and apply the properties of logarithms
- Be able to use calculators to compute logarithms

Inverses

In mathematics, **inverses** can be defined as the reverse operation (operations that revert the original operation).

$12^x = 4.4982$

We've dealt with plenty of examples of inverses in the past. Consider the following examples.

$x + 12 = 3$
$-12 \quad -12$

$2x = 6$
$/2 \quad /2$

Starting with a…

Multiplication and division: $2 \times a \div 2 = a$

Addition and subtraction: $2 + a - 2 = a$

Powers and roots: $\sqrt[2]{a^2} = a$

One-to-one Function and Inverse Function: $f^{-1}(a) = a$

The Inverse of Exponential Function

With regards to the exponential function, there exists an inverse function which is called the **Logarithmic Function**.

Definition: Let a be a number with $a \neq 1$. The **logarithmic function** with base a, denoted by $\boldsymbol{log_a}$, is defined by

$$log_a x = y \quad \text{if and only if} \quad a^y = x$$

So, here y is the *exponent* to which the base a must be raised to give x.

$12^x = 4.498$

$\log_{12} 12^x = \log_{12}(4.498$

$x =$

$11^{\log_{11} x} = 3$

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Examples

Revisiting our warm-up problems then...

1) $2^x = 8$

$x = 3$ means

$\log_2 8 = 3$

2) $4^x = 16$

$x = 2$ means

$\log_4 16 = 2$

3) $3^x = 27$

$x = 3$ means

$\log_3 27 = 3$

4) $5^x = 625$

$x = 4$ means

$\log_5 625 = 4$

5) $e^x = 1$

$x = 0$ means

$\log_e 1 = 0$

6) $10^x = 100000$

$x = 5$ means

$\log_{10} 100000 = 5$

Properties of Logarithms

We are familiar with some of the <u>properties *of exponents*</u>. Here, we've established that log_a is an exponent. Therefore, the **<u>properties of logarithms</u>** exist, much similar to the properties of exponents.

$a^0 = 1$

Property	Reason
1. $log_a 1 = 0$	Anything raised to the zero power is 1
2. $log_a a^1 = 1$	Anything raised to the 1st power is itself
3. $log_a a^x = x$	a raised to the x power is a^x
4. $a^{log_a x} = x$	$log_a x$ is the power to which a must be raised to get x

Examples

For base 5…

By property 1:

$$log_5 1 = 0$$

By property 2:

$$\cancel{log_5 5} = 1$$

By property 3:

$$\cancel{log_5 5}^8 = 8$$

By property 4:

$$\cancel{5^{log_5}}^{12} = 12$$

You try

For base 10…

By property 1:

$$log_{10}1 = 0$$

By property 2:

$$log_{10}10 = 1$$

By property 3:

$$log_{10}10^4 = 4$$

By property 4:

$$10^{log_{10}11} = 11$$

Common Logarithms

In logarithms, base 10 is considered the "standard base." Therefore, it has a special name within the logarithmic function.

<u>Common Logarithm</u>- Base 10 said to be the **<u>common base</u>**, so any log base 10 is denoted without the base written:

$$\boldsymbol{log_{10}9 = log\ 9}$$

So always assume that log has base 10 if there is no base written.

Ex. $\log 100 = 2$ and $\log 10 = 1$

In Closing

Write the answer to the following questions and share with a partner.

1. What is $log_9 81$ equal to, and why?

2. Explain why $a^{log_a x} = x$

Homework 11/12

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