Lesson 3-7

Optimization

Objective

Students will…

- Be able to solve applied minimum and maximum problems.

Guidelines for Curve Sketching

One of the most common applications of calculus involving optimization (maximizing or minimizing).

GUIDELINES FOR SOLVING APPLIED MINIMUM AND MAXIMUM PROBLEMS

1. Identify all *given* quantities and all quantities *to be determined*. If possible, make a sketch.
2. Write a **primary equation** for the quantity that is to be maximized or minimized. (A review of several useful formulas from geometry is presented inside the back cover.) * Mostly the last sentence.
3. Reduce the primary equation to one having a *single independent variable*. This may involve the use of **secondary equations** relating the independent variables of the primary equation. *Often have the quantity or answer given.
4. Determine the feasible domain of the primary equation. That is, determine the values for which the stated problem makes sense.
5. Determine the desired maximum or minimum value by the calculus techniques discussed in Sections 3.1 through 3.4.

Example

A manufacturing company wants to design an open box having a square base and a surface area of 108 in^2. What dimensions will produce a box with maximum volume.

$$\begin{array}{c|c} + & - \\ 0 \quad & 6 \quad\; 7 \end{array}$$

$V = x^2 h$

$SA = x^2 + 4xh = 108$

$-x^2 \qquad -x^2$

$\frac{\cancel{4x}h}{\cancel{4x}} = \frac{108 - x^2}{4x}$

$h = \frac{108 - x^2}{4x}$

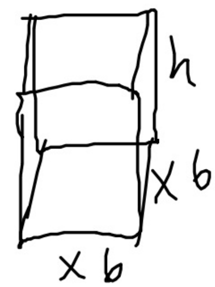
$h = \frac{108 - 36}{24} = 3$

$V(x) = \cancel{x^2}\left(\frac{108 - x^2}{4\cancel{x}}\right)$ f

$= \frac{108x - x^3}{4}$ g

$V'(x) = \frac{(108 - 3x^2)(4) - 0(\ldots)}{16} = \frac{108 - 3x^2}{4}$

$\boxed{6 \times 6 \times 3}$

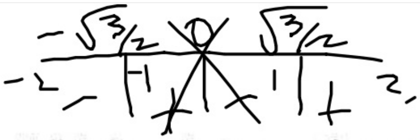


CV: $0 = 108 - 3x^2$

$\frac{-108}{-3} = \frac{-3x^2}{-3}$

$36 = x^2$

$\pm 6 = x$



Example

$y = -x^2 + 4$ min distance.

Which points on the graph of $y = 4 - x^2$ are closest to the point $(0,2)$?

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$d(x) = \sqrt{(0-x)^2 + (2-(4-x^2))^2}$$

$$= \sqrt{x^2 + (-2+x^2)^2} = (x^2+4-4x^2+x^4)^{1/2} = (-3x^2+4+x^4)^{1/2}$$

$$d'(x) = \frac{1}{2}(-3x^2+4+x^4)^{-1/2} \cdot (-6x+4x^3)$$

$$= \frac{-6x+4x^3}{2(-3x^2+4+x^4)^{1/2}}$$

CV: $0 = -6x + 4x^3$
$= 2x(-3+2x^2)$

$x = 0, \pm\sqrt{\frac{3}{2}}$

$\boxed{\left(-\sqrt{\tfrac{3}{2}}, 2.5\right) \quad \left(\sqrt{\tfrac{3}{2}}, 2.5\right)}$

Example

A rectangular page is to contain $24in^2$ of print. The margins at the top and bottom of the page are to be 1.5 inches, and the margins on the left and right are to be 1 inch. What should the dimensions of the page be so that the least amount of paper is used?

$\underbrace{\text{least amount}}_{\text{min area}}$

Print area.

$A = lw = 24$

$l = \frac{24}{w}$

$A = (l+2)(w+3)$ f g

$A(w) = \left(\frac{24}{w}+2\right)(w+3) = (24w^{-1}+2)(w+3)$

$A'(w) = -24w^{-2}(w+3) + 1(24w^{-1}+2)$

$= \cancel{-24w^{-1}} - 72w^{-2} + \cancel{24w^{-1}} + 2$

CV: $0 = -\frac{72}{w^2} + 2 \Rightarrow -2 = -\frac{72}{w^2} \Rightarrow w^2 = 36$

$w = 6$

$l = 4 + 2$

$w = 6 + 3$

9 by 8

Example

Two posts, one 12 ft high and the other 28 ft high, stand 30 ft apart. They are to be stayed by two wires, attached to a single stake, running from ground level to the top of each post. Where should the stake be placed to use the least amount of wire?

Perimeter + Circumference

Example

$\frac{1}{\pi+4}$

0 — | + 1

Four feet of wire is to be used to form a square and a circle. How much of the wire should be used for the square and how much should be used for the circle to enclose the maximum total area?

$$4 = 4x + 2\pi r \qquad A = x^2 + \pi r^2$$

$$\frac{4-4x}{2\pi} = \frac{\cancel{2\pi} r}{\cancel{2\pi}}$$

$$A(x) = x^2 + \pi\left(\frac{4-4x}{2\pi}\right)^2 = x^2 + \cancel{\pi}\left(\frac{16-8x+16x^2}{4\pi^{\cancel{2}}}\right)$$

$$\frac{\pi}{\pi}x^2 + \left(\frac{4-2x+4x^2}{\pi}\right) = \frac{\pi x^2 + 4 - 2x + 4x^2}{\pi}$$

$$A'(x) = \frac{(2\pi x - 2 + 8x)\cancel{\pi}}{\pi^{\cancel{2}}} = \frac{2\pi x - 2 + 8x}{\pi}$$

CV: $2\pi x - 2 + 8x = 0$

$$\Rightarrow (2\pi + 8)x - 2 = 0$$

$$(2\pi + 8)x = 2$$

$$x = \frac{2}{2\pi + 8}$$

CV: $\frac{1}{\pi+4}$

$\frac{4}{\pi+4}$ - square

$\frac{4 - \frac{4}{\pi+4}}{2\pi}$ - circle

Homework 11/28

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