Lesson 3-5

Complex Zeros II

Objective

Students will…

- Be able to find polynomials with specified zeros.
- Be able to understand what Conjugate Zeros Theorem says and use it to find polynomials.

Finding Polynomials with Specified Zeros

We have been learning how to factor a polynomial in order to find its zeros. The backward process can also be done. Consider a polynomial *P* with zeros 0 and -3. Based on what we know about factored polynomials. These zeros can be derived from $P(x) = x(x + 3)$

So, when we multiply it out, $P(x) = x^2 + 3x$

$Q(x) = 2x^2 + 6x$

Hence, if the degree of the polynomial is known, along with its zeros, we can derive the original function.

Example

Find a polynomial $Q(x)$ of degree 4, with zeros -2 and 0, where -2 is a zero of multiplicity 3. **Note:** $(A+B)^3 = (A^3 + 3A^2B + 3AB^2 + B^3)$

$$Q(x) = x(x+2)^3 = x\left(x^3 + 3x^2(2) + 3x(2)^2 + 2^3\right)$$

$$= x(x^3 + 6x^2 + 12x + 8)$$

$$= \boxed{x^4 + 6x^3 + 12x^2 + 8x}$$

Example

Find a polynomial $P(x)$ of degree 4, with zeros $i, -i, 2$, and -2.

$$P(x) = (x-i)(x+i)(x-2)(x+2)$$

$$= (x^2 - \overset{-1}{i^2})(x^2-4)$$

$$= (x^2+1)(x^2-4) = \boxed{x^4 - 3x^2 - 4}$$

Conjugate Pairs

There is an interesting thing to observe regarding conjugates of complex numbers.

<u>**Conjugate Zeros Theorem-**</u> If the polynomial P has real coefficients, and if the complex number z is a zero of P, then its complex conjugate is also a zero of P.

In other words, if a certain $a + bi$ is a zero (x-intercept) of a polynomial, then its conjugate, $a - bi$ is also a zero.

Example

Find a polynomial $P(x)$ of degree 3 that has integer coefficients and zeros $\frac{1}{2}$ and $3 - i$. , $3+i$

$$P(x) = \left(x - \frac{1}{2}\right)(x - 3 + i)(x - 3 - i)$$

$$= (2x - 1)(x^2 - 3x - \cancel{ix} - 3x + 9 \cancel{+ 3i} + \cancel{ix} - \cancel{3i} + 1)$$

$$= (2x - 1)(x^2 - 6x + 10) = 2x^3 - 12x^2 + 20x - x^2 + 6x - 10$$

$$= \boxed{2x^3 - 13x^2 + 26x - 10}$$

Homework 10/28

TB pg. 298 #31-37 (odd), 41, 46