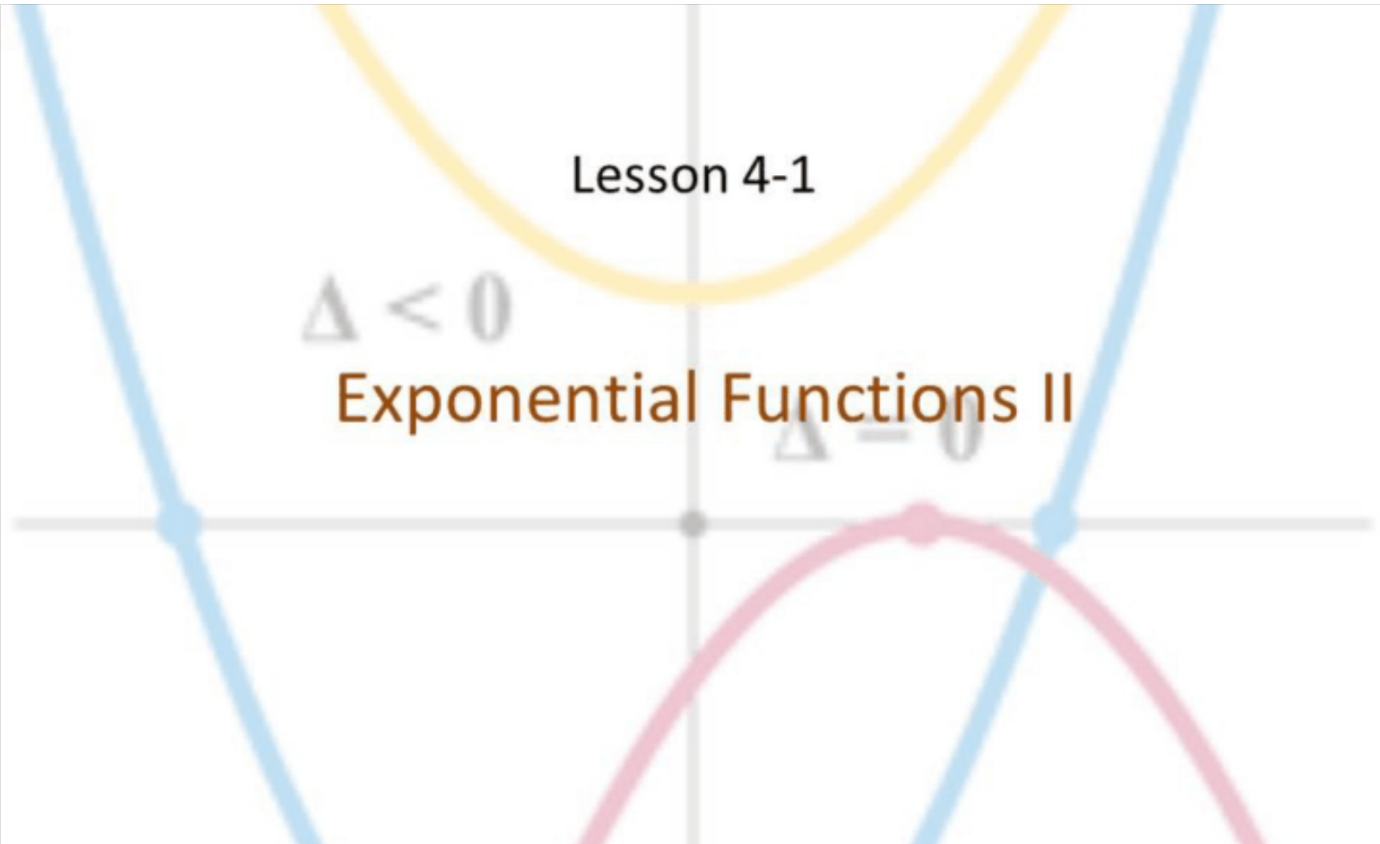
Lesson 4-1

Exponential Functions II



Objective

Students will…

- Be able to identify the end behavior of exponential functions.
- Be able to derive the exponential function of whose graph is given.

Exponential Functions

In our previous chapter, we studied polynomial and rational functions. Yet another important and practical function group is the exponential function.

$1^x = 1$

The **exponential function** with **base** a is defined for all real numbers by

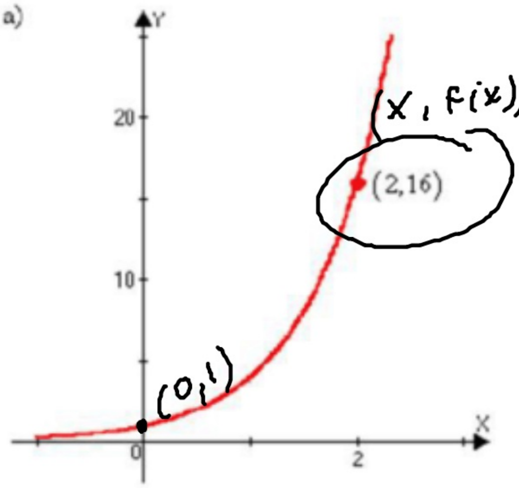
$f(x) = a^x$, where $a > 0$ and $a \neq 1$.

Also, note that here our **exponent** is the variable, instead of the **base**.

Deriving Exponential Functions

We can find the equation of the functions from the given graphs. The idea is to use the exponential definition, $f(x) = a^x + k = c\,a^{(x-h)} + k$

Ex. a)



$\sqrt{16} = \sqrt{a^2}$

$\pm 4 = a$

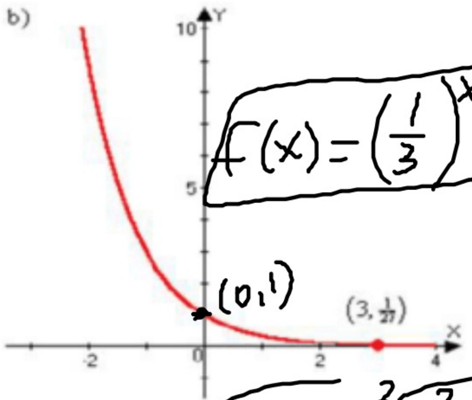
$f(x) = 4^x$

Examples

$\frac{1^x}{3} \neq \frac{1}{3}^x$

Find the exponential function $f(x) = a^x + k$ whose graph is given.

1.

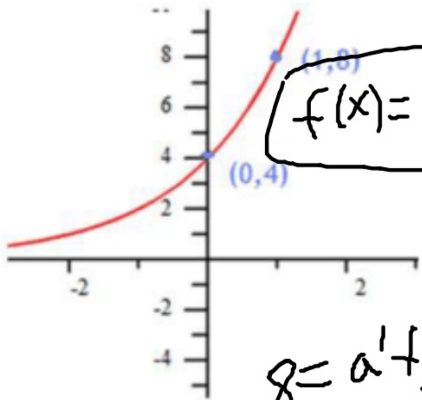


$f(x) = \left(\frac{1}{3}\right)^x$

$\sqrt[3]{\frac{1}{27}} = \sqrt[3]{a^3}$

$\frac{1}{3} = a$

2.



$f(x) = 5^x + 3$

$8 = a^1 + 3$

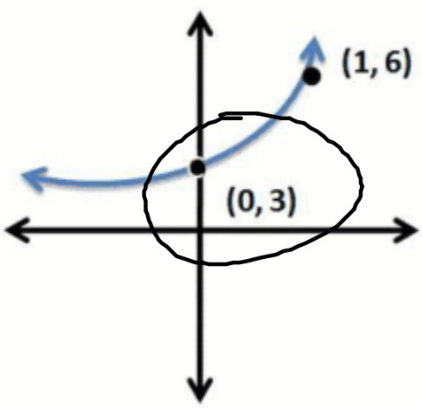
$-3 \qquad -3$

$5 = a$

Examples

Find the exponential function $f(x) = a^x + k$ whose graph is given.

3.



$6 = a^1 + 2$

$4 = a$

$f(x) = 4^x + 2$

The *Natural* Exponential Functions

In studying exponential functions, there is a very special number that is studied, mainly because of its use virtually on a daily basis out in the real world. It is a called the *Natural* exponential function, denoted as e

So, by definition, the ***natural* exponential function** is the exponential function

$$f(x) = e^x, \text{ where the base } e \approx 2.71828 \ldots$$

By definition e is the value that $\left(1+\frac{1}{n}\right)^n$ approaches as $n \to \infty$

This will be studied much more extensively in Calculus. For this course, our focus is simply using this strange number via a calculator ☺

"e" "x y"
"e x"

Examples

Evaluate each expression correct to five decimal places.

1. e^3 ≈ 20.08554

2. $2e^{-0.53}$ ≈ 1.17721

3. $e^{4.8}$ ≈ 121.51042

Compound Interest

The question was why use such a strange and random base? Actually, it turns out that this little e has much use out in the real world. Again, in Calculus you will see that e isn't all that "random," and have a better idea why e has so much use out in the real world. Here's an example: Calculating Compound Interest! Ka-ching!

Compound Interest is calculated by the formula $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$,

where $A(t) =$ the amount after t years

$P =$ the Principal amount (initial amount put in)

$r =$ the interest rate per year

$n =$ the number of times interest[redacted] is compounded per year

$t =$ the number of years

Example

A sum of $1000 is invested at interest rate of 12% per year. Find the amounts in the account after 3 years if interest is compounded annually. Quarterly?

Here we need to use our compound interest formula.

$=P$, $=0.12=r$, $=t$, $n=1$, $n=4$

3.229

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$

Annually: $A(3) = 1000\left(1+\frac{0.12}{1}\right)^{(1\cdot 3)} \approx 1404.92$

Quarterly: $A(3) = 1000\left(1+\frac{0.12}{4}\right)^{(4\cdot 3)} \approx 1425.76$

Continuously Compounded Interest

For some accounts, the interest is compounded continuously, rather than periodically (like our previous problem). For this kind of compounding, the formula is actually much simpler.

Continuously Compounded Interest is calculated by,

$$A(t) = Pe^{rt} \quad \text{where,}$$

$A(t) =$ the amount after t years

$P =$ the Principal amount (initial amount put in)

$r =$ the interest rate per year

$t =$ the number of years

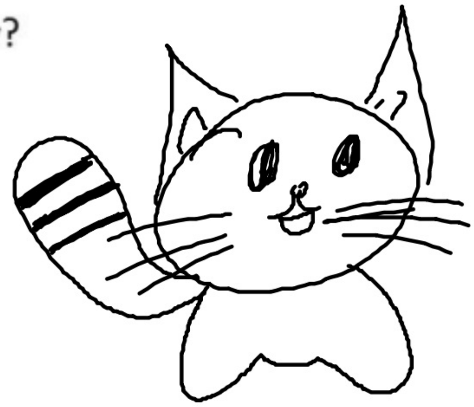
Example

Find the amount after 3 years if $1000 is invested at an interest rate of 12% per year, compounded continuously.

$$A(3) = 1000\, e^{(0.12 \cdot 3)} \approx 1433.32$$

A sum of $3000 is invested. What is the balance in the account and the amount of interest after 4 years if you earn:

a. 1.9% interest compounded annually?



b. 1.6% compounded monthly?

A sum of $3000 is invested. What is the balance in the account and the amount of interest after 4 years if you earn:

c. 1.4% compounded daily?

d. 0.765% compounded continuously?

Homework 11/17

TB pg. 336-337 #15-24, 39, 40, 75, 77