Lesson 4-1

Exponential Functions III

Objective

Students will…

- Be able to solve (compounded)interest problems using the natural exponential function, $f(x) = e^x$

Exponential Functions

In our previous chapter, we studied polynomial and rational functions. Yet another important and practical function group is the exponential function.

The **exponential function** with **base** a is defined for all real numbers by

$$f(x) = a^x, \text{ where } a > 0 \text{ and } a \neq 1.$$

Also, recall the ***natural* exponential function**, which is the exponential function

$$f(x) = e^x, \text{ where the base } e \approx 2.71828 \ldots$$

Compound Interest

The question was why use such a strange and random base? Actually, it turns out that this little e has much use out in the real world. Again, in Calculus you will see that e isn't all that "random," and have a better idea why e has so much use out in the real world. Here's an example: Calculating Compound Interest! Ka-ching!

Compound Interest is calculated by the formula: $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$,

where $A(t) =$ the amount after t years

$P =$ the Principal amount (initial amount put in)

$r =$ the interest rate per year /100 -

$n =$ the number of times interested is compounded per year

$t =$ the number of years

Example

A sum of \$1000 is invested at interest rate of 12% per year. Find the amounts in the account after 3 years if interest is compounded annually. Quarterly?

Here we need to use our compound interest formula.

P, $r = 0.12$, t, $n = 1$

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$

$$A(3) = 1000\left(1+\frac{0.12}{1}\right)^{(1\cdot 3)}$$

$$= \$1404.92$$

$$A(t) = 1000\left(1+\frac{0.12}{4}\right)^{(4\cdot 3)}$$

$$\approx \$1425.76$$

Continuously Compounded Interest

For some accounts, the interest is compounded continuously, rather than periodically (like our previous problem). For this kind of compounding, the formula is actually much simpler.

Continuously Compounded Interest is calculated by,

$A(t) = Pe^{rt}$ where,

$A(t) =$ the amount after t years

$P =$ the Principal amount (initial amount put in)

$r =$ the interest rate per year

$t =$ the number of years

Example

Find the amount after 3 years if $1000 is invested at an interest rate of 12% per year, compounded continuously.

$A(t) = 1000e^{(0.12 \cdot 3)} \approx \$1433.32$

A sum of $3000 is invested. What is the balance in the account and the amount of interest after 4 years if you earn:

a. 1.9% interest compounded annually?

3234.58

$3000\left(1+\frac{0.019}{1}\right)^{(1\cdot 4)}$

b. 1.6% compounded monthly?

$3198.14

A sum of $3000 is invested. What is the balance in the account and the amount of interest after 4 years if you earn:

c. 1.4% compounded daily?

$3172.78

d. 0.765% compounded continuously?

3093.71

In Closing

Explain to your neighbor the difference between continuous and compound interest.

Homework 11/7

Compound Interest WKSHT