Lesson 5-1, 5-4a

Exponential and Logarithmic Functions and their Derivatives I

Objective

Students will…

- Be able evaluate the derivatives of exponential functions.
- Be able to evaluate the derivatives of logarithmic functions.

Natural Exponential Functions

Recall, the number e. We define the natural exponential function as $f(x) = e^x$, where $e \approx 2.718281828459...$

The derivative of the natural exponential function is a simple chain rule:

$$\frac{d}{dx}[e^u] = e^u(u')$$

In other words, the derivative of e^u is e^u times the derivative of u.

Example

Find the derivative of the following:

a. $f(x) = e^{x-1}$

$f'(x) = e^{x-1} \cdot 1$

$= \boxed{e^{x-1}}$

b. $y = e^{1-x}$

$y' = e^{1-x} \cdot -1$

$= \boxed{-e^{1-x}}$

c. $g(x) = e^{2x} = (e^{x})^{2}$

$g'(x) = e^{2x} \cdot 2$

$= \boxed{2e^{2x}}$

Example

Find the derivative of the following:

d. $f(x) = (e^x)^2$

$f'(x) = 2(e^x) \cdot e^x$

$= \boxed{2e^{2x}}$

e. $y = e^{2x}e^x$

$y = e^{2x+x} = e^{3x}$

$y' = e^{3x} \cdot 3$

$= \boxed{3e^{3x}}$

f. $y = e^{198}$

$y' = \boxed{0}$

$y' = e^{198} \cdot 0$

$= \boxed{0}$

$\ln x = \log_e x$ Natural Logarithmic Functions

Recall that the inverse function of e^x is known as the natural logarithmic function, or namely, $f(x) = \ln x$.

"The answer to the log is the exponent"

Remember, $b = e^a \quad \rightarrow \quad \ln b = a$

The derivative of the natural log function is also a chain rule as follows:

$$\frac{d}{dx}[\ln u] = \frac{1}{u}(u') = \frac{u'}{u}$$

So the derivative of $\ln u$ is 1 over u times the derivative of u.

$\boxed{\frac{d}{dx}[\ln ax]} = \frac{1}{x}$

Example

Find the derivative of the following:

a. $f(x) = \ln 2x$

$f'(x) = \frac{1}{2x} \cdot 2$

$= \frac{\cancel{2}}{\cancel{2}x} = \frac{1}{x}$

$f(x) = \ln 2 + \ln x$

$f'(x) = 0 + \frac{1}{x}$

b. $y = \ln(x^2 + 1)$

$y' = \frac{1}{x^2+1} \cdot 2x$

$= \boxed{\frac{2x}{x^2+1}}$

Example

c. $g(x) = \overset{f}{x} \overset{g}{\ln x}$

$\cancel{= \ln x^x}$

$g'(x) = 1\ln x + \frac{1}{x} \cdot x$

$= \ln x + \frac{x}{x}$

$= \ln x + 1$

d. $y = \ln(x^3)$

$y' = \frac{1}{x^3} \cdot 3x^2 = \frac{3\cancel{x^2}}{\cancel{x^3}} = \boxed{\frac{3}{x}}$

$y = 3\ln x$

$y' = 3\left(\frac{1}{x}\right) = \boxed{\frac{3}{x}}$

Example

e. $y = (\ln x)^3$

$= (\ln x)(\ln x)(\ln x)$

$y' = 3(\ln x)^2 \cdot \frac{1}{x}$

$= \boxed{\frac{3(\ln x)^2}{x}}$

f. $y = \ln 4x + \ln 2x - \ln 8x = \ln\left(\frac{4x \cdot 2x}{8x}\right.$

$y' = \frac{1}{\cancel{4}x} \cdot \cancel{4} + \frac{1}{\cancel{2}x} \cdot \cancel{2} - \frac{1}{\cancel{8}x} \cdot \cancel{8}$

$= \frac{1}{x} + \frac{1}{x} - \frac{1}{x} = \boxed{\frac{1}{x}}$

Laws of Exponents/Logarithms Review

Laws of Exponents

1. $x^a x^b = x^{a+b}$
2. $\frac{x^a}{x^b} = x^{a-b}$, given that $x^b \neq 0$
3. $(x^a)^b = x^{ab}$

Laws of Logarithms:

1. $log_a(AB) = log_a A + log_a B$
2. $log_a\left(\frac{A}{B}\right) = log_a A - log_a B$
3. $log_a(A^C) = C log_a A$

Homework 11/15

5.1 #19-33 (odd), 45-67 (e.o.o)

5.4 #35-45 (odd)