Lesson 4-4

Fundamental Theorem Of Calculus

Objective

Students will…

- Be able to know the Fundamental Theorem of Calculus.
- Be able to use the FTC to find the area underneath the curve.

The Velocity Problem

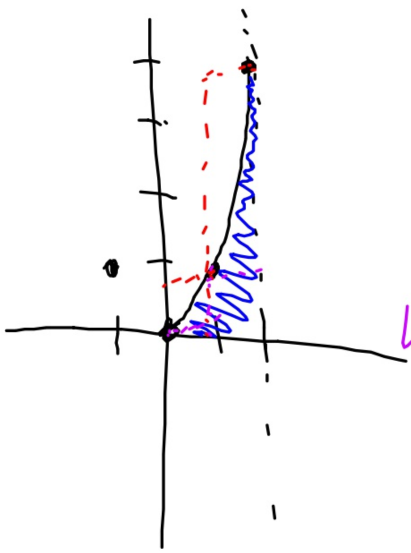
$F(x)$ = Total Displacement $f'(x)$ = Velocity

$F(x) = \frac{1}{3}x^3 + C$. $f''(x)$ = acceleration.

Consider the position function $f(x) = x^2$. (positive Domain).

$f'(x) = 2x$.

$f''(x) = 2$



R sum: $1 + 4 = 5$

L sum: $0 + 1 = 1$

$1 \leq \text{Area} \leq 5$

$F(x) = \frac{1}{3}x^3$

$F(2) = \frac{1}{3}(2)^3 = \frac{8}{3} = 2\frac{2}{3} \approx 2.67$

$- \quad F(0) = \frac{1}{3}(0)^3 = 0$

Exact Area: $\rightarrow \frac{8}{3} \approx 2.67$

Fundamental Theorem of Calculus

Fundamental Theorem of Calculus- If a function f is continuous on the closed interval $[a, b]$, then $\int_a^b f(x)\,dx = F(b) - F(a)$ (Def. Definite Int.)

upper - lower.

Recall that $F(x)$ is the **antiderivative** of $f(x)$.

This is clearly the most important theorem of all of Calculus (hence the word "fundamental"). Provided you can find an antiderivative of f, you now have a way to evaluate the area underneath the curve (instead of approximating) using **definite integral**.

Example

Use five rectangles to find the area of the region bounded by $f(x) = x^2$, the x-axis and $x = 0$ and $x = 10$.

$$\int_0^{10} x^2\,dx = \frac{1}{3}x^3 \Big|_0^{10} = \frac{1}{3}(10)^3 - \frac{1}{3}(0)^3 = \frac{1000}{3} - 0 = \boxed{\frac{1000}{3}}.$$

Example

Use five rectangles to find the area of the region bounded by $f(x) = -x^2 + 5$, the x-axis and $x = 0$ and $x = 2$.

$$\int_0^2 -x^2+5\,dx = -\frac{1}{3}x^3+5x \Big|_0^2 = \left(-\frac{1}{3}(2)^3+5(2)\right) - 0$$

$$= -\frac{8}{3} + \frac{30}{3} = \boxed{\frac{22}{3}}$$

Example

Use six rectangles to find the area of the region bounded by $f(x) = \sin x$, the x-axis and $x = 0$ and $x = \pi$

$$\int_0^{\pi} \sin x \, dx = -\cos x \Big|_0^{\pi} = -\cos \pi + \cos(0)$$

$$= 1 + 1 = \boxed{2}$$

Homework 1/17

~~Previous~~ WKSHT use the definite integral to evaluate the actual area.