

6). $-4, -\frac{8}{3}, -2, -\frac{8}{5}, \dots$

Not Arithmetic

13) $3, 6, 12, 24, \dots$
 $\times 2 \times 2 \dots$

$f(n) = \frac{3}{2}(2^n)$

10). $\frac{30}{3}, \frac{28}{3}, \frac{26}{3}, \frac{24}{3}, \dots$

$f(x) = -\frac{2}{3}x + \frac{32}{3}$

Convergence/Divergence.

ex. 0, 1, 2, 3, 4, 5, 6, ...

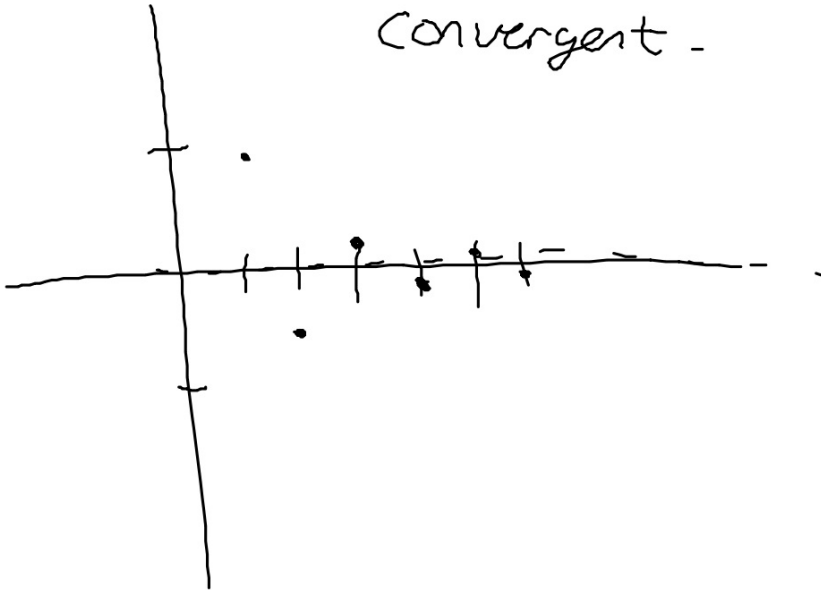
divergent.

n	1	2	3	4	...
ex. f(n)	$\frac{1}{1}$	$-\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{4}$	$\frac{1}{5}$, $-\frac{1}{6}$, ...

$$f(n) = (-1)^{n+1} \frac{1}{n}$$

$$f(n) = (-1)^{n-1} \frac{1}{n}$$

Convergent.



Convergent expressions ($n \rightarrow \infty$)

a) $\frac{1}{n} = \frac{1n^0}{n^1}$

b) $\frac{n^a}{n^b}$ given $b > a$.

c) $\frac{cn^a + \dots}{dn^a + \dots}$

Converges to $\frac{c}{d}$

ex. $\frac{n^3 + 4}{n^2 + 4n + 6}$

ex. $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \dots$

$\Rightarrow f(n) = \frac{1}{n^2}$ Convergent.

ex. $(1, \frac{4}{2}, \frac{9}{3}, \frac{16}{4}, \frac{25}{5}, \dots)$

$= 1, 2, 3, 4, \dots$
 $f(n) = \frac{n^2}{n}$ divergent.

ex. $(\frac{1}{1}, \frac{4}{8}, \frac{1}{27}, \frac{16}{64}, \frac{25}{125}, \dots)$

$f(n) = \frac{n^2}{n^3}$ Convergent

Determine if the following functions are convergent. If so, then find their value of convergence. \rightarrow Limit of a function.

a) $\frac{14}{n}$
 $= 14 \frac{1}{n}$

$C; 0$

b) $\frac{12n^3}{10000000n^2}$

D

c) $\frac{n^{19} + n^{18} + 4n^{14} + 12n^{11} + 146}{n^{21} + 3}$

$C; 0$

d) $\frac{(n^2 + \dots)(n+3)(n+31)}{2n^2 + 3}$

$C; \frac{1}{2}$

e) $\frac{2en^4 + 3n^2 + \pi}{(2\pi n^2 + 3)(en^2 + 3)}$

$4\pi en^4 + \dots$
 $= \frac{12e}{24\pi e} \cdot \frac{1}{2\pi}$

Limits

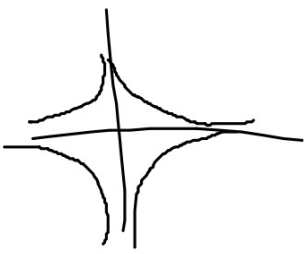
$$\lim_{n \rightarrow \infty} \frac{1}{n} = 0 \quad C$$

$$\lim_{n \rightarrow 0} n \quad C$$

$$\lim_{n \rightarrow 0} \frac{1}{n} = \infty \quad D$$

$$\lim_{n \rightarrow \infty} n \quad D$$

$$\lim_{n \rightarrow -\infty} \frac{1}{n} = 0 \quad C$$



Sucks

$\lim_{n \rightarrow \infty} \rightarrow \lim_{n \rightarrow -\infty}$
Not opposites.

mhpfndteswraeF
'LO{ -
p()p/9;87uk45j3tw
5h3qre Ca

$$7) \begin{matrix} \cancel{6} & \cancel{1} & \cancel{4} & \cancel{9} \\ \cancel{3} & /3 & /3 & /3 \\ -5/3 & -5/3 & -5/3 & \end{matrix}$$

$$\begin{matrix} -12 & -8 & -6 & -8 \\ \cancel{4} & /3 & \cancel{2} & /5 \\ \cancel{3} & \times 1/3 & \cancel{1} & \times 2/3 \end{matrix}$$

$$f(x) = -5/3x + 1/3$$

- Not Arithmetic.

Convergence/Divergence

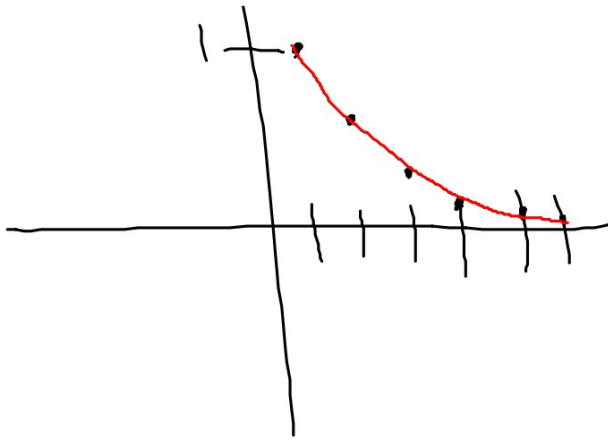
ex. $1, 2, 3, 4, 5, 6, 7, \dots \rightarrow \infty$, diverges.

$$f(n) = n$$

n | 2 3 4 5 6 ... Converges to 0.

$$f(n) = \frac{1}{n}$$

$$f(n) = \frac{1}{n}$$



Limit

Convergence - A sequence is said to be convergent if there exists a certain limit ^(other than ∞). If a sequence does not converge, it is said to diverge.

ex. $n=1, 2, 3, 4, 5, \dots$
 $f(n) = \frac{1}{n}$ \Rightarrow Converging to 0.
 $\Rightarrow \boxed{\lim_{n \rightarrow \infty} \frac{1}{n} = 0}$

The limit of the function $\frac{1}{n}$ as n approaches infinity is 0.

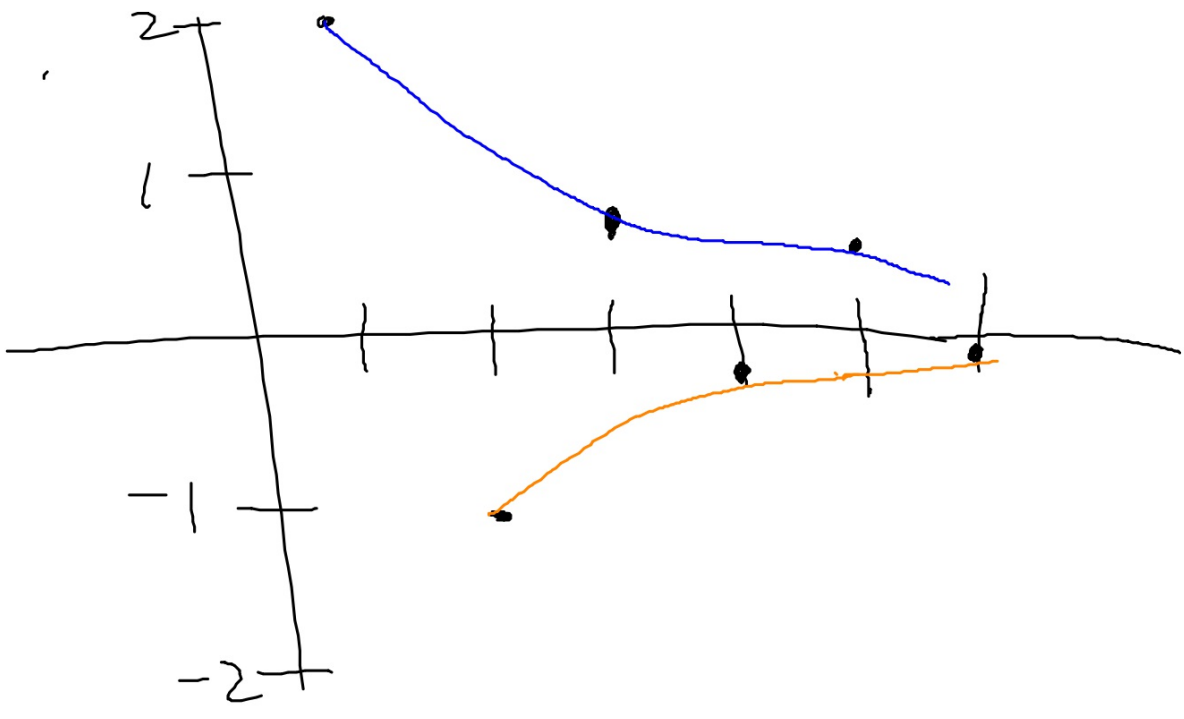
ex. $1, 2, 3, 4, 5, 6, \dots$
 $f(n) = n$ $\Rightarrow \boxed{\lim_{n \rightarrow \infty} n = \infty}$

ex. $2, -1, \frac{2}{3}, \frac{-1}{2}, \frac{2}{5}, \frac{-1}{3}, \frac{2}{7}, \dots$

$(\frac{1}{n})$

$f(n) = (-1)^{n+1} (\frac{2}{n})$

Converging to $0 \Rightarrow \lim_{n \rightarrow \infty} (-1)^{n+1} (\frac{2}{n}) = 0$



Convergent expressions $n \rightarrow \infty$.

a) $\frac{1}{n}$ Converges to 0

b) $\frac{n^a}{n^b}$ given $b > a$ Converges to 0.

c) $\frac{cn^a}{dn^a}$ Converges to $\frac{c}{d}$.

$n \rightarrow \infty$ ex. $\frac{3n^7}{1000n^3 + 1034n^3 + 111n^2 + 24087}$

Determine whether the following functions are convergent ($n \rightarrow \infty$).
If they are, find their limit.

$$1) \frac{14}{n} \\ = 14 \left(\frac{1}{n} \right) \subset ; 0$$

$$2) \frac{n^3}{n} = \frac{n^2}{1} = n^2 \quad 3) \frac{47289n^{\cancel{4}}}{100000000\pi} \quad \text{D}$$

$$4) \frac{2n^4 + 3}{3n^4 + 4n^2}$$

$$\subset ; \frac{2}{3}$$

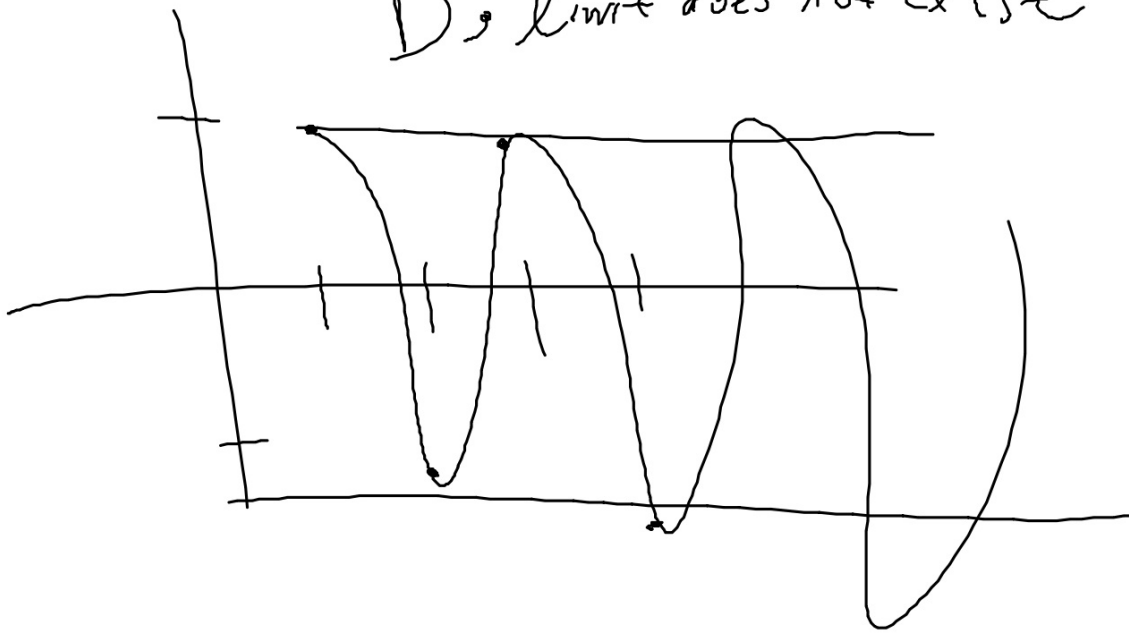
$$5) \frac{2e^{\pi} n^{27} + 9n^2 + 4n + 104742}{4en^{27} + 1000042492}$$

$$\subset ; \frac{18e^{\pi}}{24e} = \frac{\pi}{2}$$

1, -1, 1, -1, 1, -1, 1, -1

$$(-1)^{n+1}$$

D; limit does not exist



$$5) -4, \frac{-8}{3}, \frac{-12}{3}, \frac{-16}{3}$$

$$-12 + \frac{4}{3} \quad + \frac{4}{3}$$

Not Arithmetic

$$12) -28, -34, -40, -46$$

$$-6 \quad -6 \quad -6$$

$$f(n) = -6n - 22$$

$$10) \frac{30}{3}, \frac{28}{3}, \frac{26}{3}, \frac{24}{3}$$

$$-2/3 \quad -2/3$$

$$f(n) = -\frac{2}{3}n + \frac{32}{3}$$

$$\lim_{n \rightarrow \infty} n = \infty$$

Convergence/Divergence

$$\lim_{n \rightarrow \infty} \frac{n^3 - 4n^2}{n^2 + 4}$$

for Blake only

ex. 1, 2, 3, 4, 5, 6, 7, ... $\rightarrow \infty$

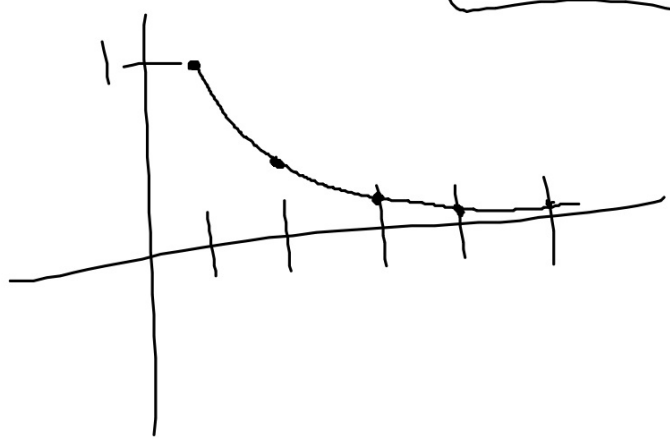
$f(n) = n$ divergent.

ex. $f(n) = (\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots)$... Converging to 0.

$$f(n) = \frac{1}{n}$$

$$\lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

Limit \rightarrow Approachment



Convergence - A sequence is said to converge if it approaches a certain number, i.e. a limit other than ∞ exists. A sequence diverges if it doesn't converge.

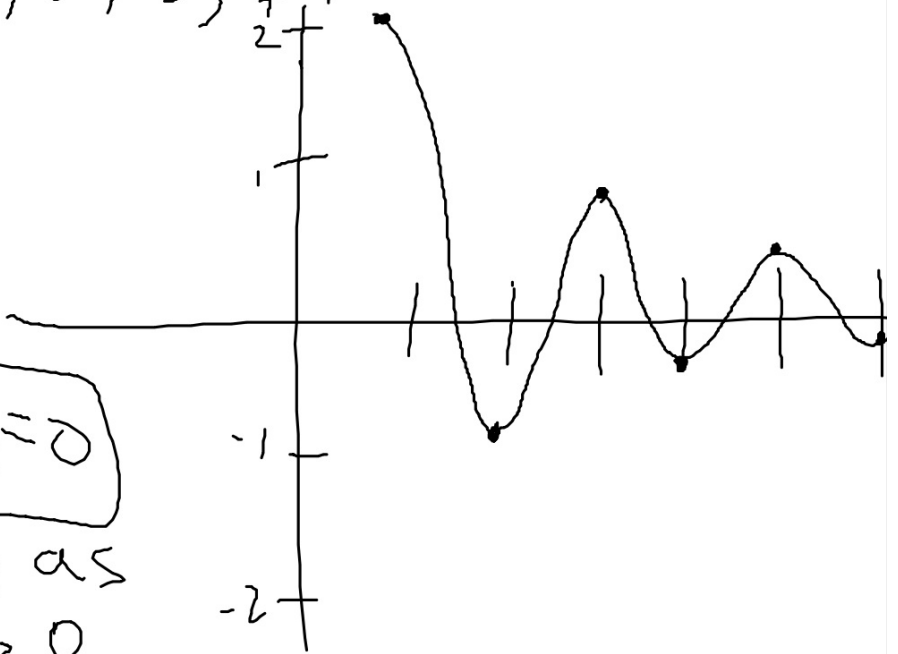
ex. $f(n)$ 1, -1, $\frac{2}{3}$, $-\frac{1}{2}$, $\frac{2}{5}$, $-\frac{1}{3}$, $\frac{2}{7}$, 1, ...

$$f(n) = (-1)^{n+1} \left(\frac{2}{n} \right)$$

Converging to 0

$$\Rightarrow \lim_{n \rightarrow \infty} (-1)^{n+1} \left(\frac{2}{n} \right) = 0$$

Limit of $(-1)^{n+1} \left(\frac{2}{n} \right)$ as n approaches ∞ is 0.



Convergent Expressions ($n \rightarrow \infty$).

a) $\frac{1}{n}$ (harmonic sequence) converges to 0.

b) $\frac{n^a + \dots}{n^b + \dots}$ given $b > a$. Converges to 0.

c) $\frac{c n^a + \dots}{d n^a + \dots}$ converges to $\frac{c}{d}$.

$\frac{3n^4}{19826n^4 + 4000n^2}$

$\frac{n^4}{n^4 + 3n^3 + 4n} \Rightarrow 1$

Determine whether the following functions are convergent. If so, find their limit. (Note: $n \rightarrow \infty$)

$$1) \frac{14}{n} = 14\left(\frac{1}{n}\right)$$

C ; 0

$$2) \frac{3n^4 + 4}{2n^4 - 2n^3 + 1}$$

C ; $\frac{3}{2}$

$$3) \frac{1/n^7}{6n^4 - 10000n^2}$$

D ; ∞

$$4) \frac{14987421n^{19}}{2n^{20}}$$

C ; 0

$$5) \frac{2e^{\pi n^2} + 4n^2 + 11}{4e^{2n} + 11111111n^{11}}$$

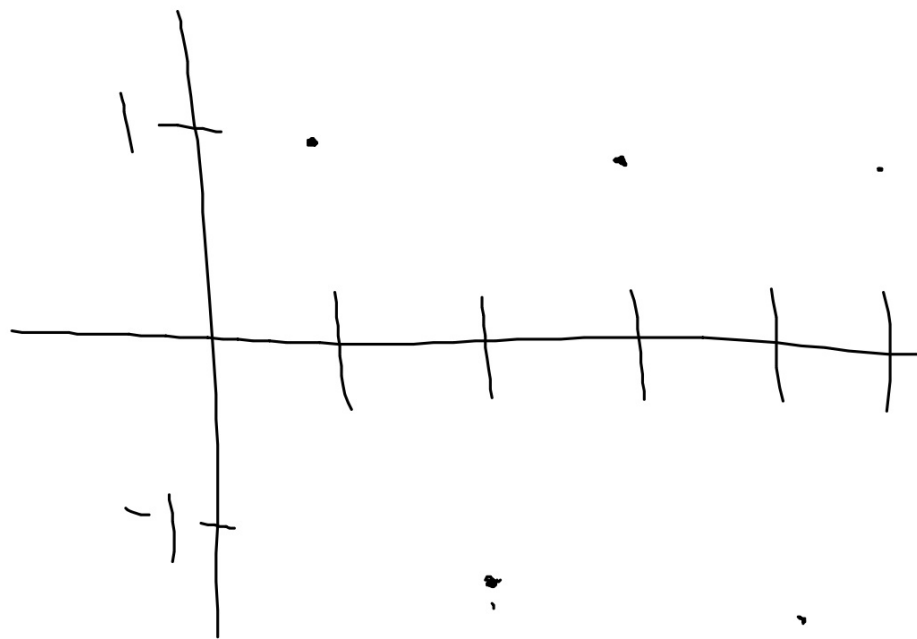
$$C ; \frac{18e^{\pi}}{24e} = \frac{\pi}{2}$$

~~$$\frac{2e^{\pi n^2}}{4e^{2n}} = \frac{e^{\pi n^2 - 2n}}{2}$$~~

ex. $-1, 1, -1, 1, -1, 1, -1, 1, \dots$

$$f(n) = (-1)^n$$

Divergent ; Limit does not exist.



$$\lim_{n \rightarrow \infty} \frac{1}{n} = 0 \quad C$$

$$\lim_{n \rightarrow -\infty} \frac{1}{n} = 0 \quad C$$

$$\lim_{n \rightarrow 0} \frac{1}{n} = \infty \quad D$$

$$\frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{9}, \dots$$

$$\frac{1}{0.000001} = 1,000,000$$

$$\lim_{n \rightarrow 3} \frac{1}{n} = \frac{1}{3}$$

