3) A die is rolled. Find the probability . . . $S = \{1, 2, 3, 4, 5, 6\}$

a) # is 6 . . . $\Rightarrow$ $1/6$

b) $3/6$.

c) $1/6$.

2). Coin tossed, die rolled.

a) $S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$

b) $3/12$

c) $2/12$

d) $3/12$

7) ball drawn from a jar with 5 red balls, 2 white balls, and one yellow.

a) red ball drawn; $5/8$

b) $7/8$

c) $0/8$

8a $5/8$

b $8/8$

c $6/8$.

Compound Events

"$\cup$" – union, i.e. "or"

"$\cap$" – Intersection, i.e. "and"

Mutually Exclusive Events – Two events that have no outcome in common.

If E and F are mutually exclusive events in a sample space S, then the probability of E or F is

$$P(E \overset{\text{"or"}}{\cup} F) = P(E) + P(F)$$

$$\Rightarrow P(E_1 \cup E_2 \cup E_3 \ldots) = P(E_1) + P(E_2) + \ldots$$

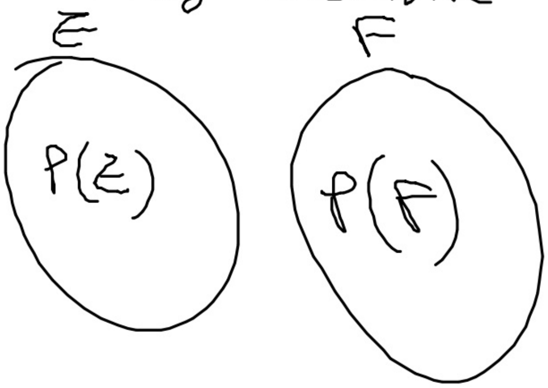
Non-Mutually Exclusive Events

If E and F are events in a sample space S, (non-mut. exc.)
then the probability of E or F is . . .

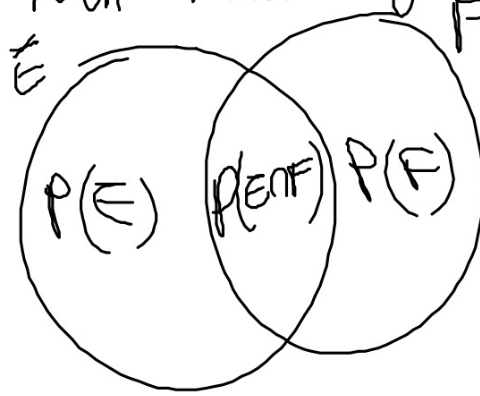
"and"

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

Mutually Exclusive



Non-Mutually Exclusive



Let event E : 7 card

event F : face card mut. exc.

$$P(E \overset{\cup}{\text{or}} F) = P(E) + P(F)$$

$$= \frac{4}{52} + \frac{12}{52}$$

$$= \boxed{\frac{16}{52}}$$

E : Face card Not

F : Spade mut. exc

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$= \frac{12}{52} + \frac{13}{52} - \frac{3}{52}$$

$$= \boxed{\frac{22}{52}}$$

Independent vs Dependent Events

Independent Events – Events in which their occurrences do not affect each other.

Ex. Tossing a coin multiple times

Probability of the intersection of independent Events.

If E and F are independent events in a sample space S, then,

$$P(E \overset{\text{"and"}}{\cap} F) = P(E)P(F)$$

Probability of non-independent events

If E and F are non-independent events in the sample space S, then..

"given"

$$P(E \cap F) = P(E)\,P(F|E)$$

<u>Ex.</u> 52-card deck. E: drawing a 6 of ♡ F: drawing a red card (w/o replacement)

$$P(E \cap F) = \frac{1}{52} \cdot \frac{25}{51} = \frac{25}{\cancel{51}}$$

2) Coin tossed, die rolled.

a) $S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$.

b) $\frac{3}{12}$

c) $\frac{2}{12}$

d) $\frac{3}{12}$

5) a card drawn from 52-card deck.

a) a King: $\frac{4}{52}$

b) face card: $\frac{12}{52}$

c) |-face card: $\frac{40}{52}$

Compound Events

"or"

$\cup$ — union

$\cap$ — Intersection

"and"

Mutually Exclusive Events.

Two or more events that have no shared outcomes.

If E and F are mutually exclusive events in sample space S, then the probability of E or F is. . .

$$P(E \cup F) = P(E) + P(F)$$

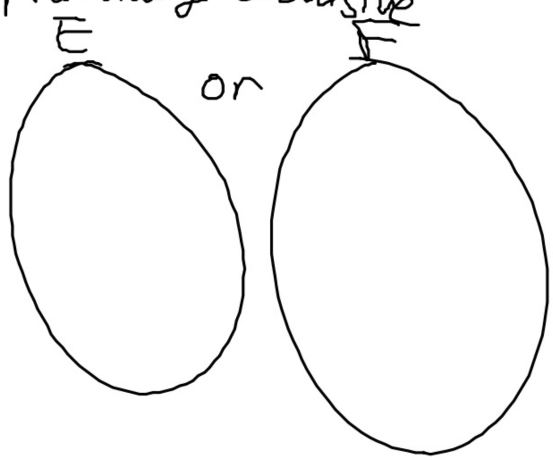
$$P(E \cup F \cup G \cup H \ldots) P(E) + P(F) + P(G) + \ldots$$

Non-mutually Exclusive Events

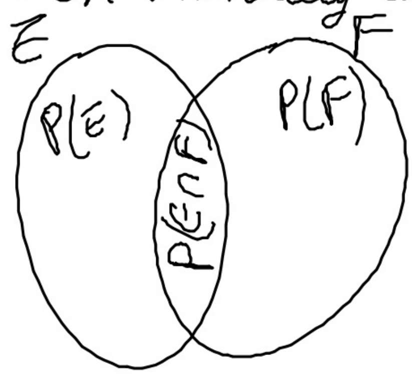
If E and F are non-mutually exclusive events in the sample space S, then...

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

Mutually Exclusive



Non-mutually Exclusive



Let E: drawing a 7
F: drawing a face card   Mutuall Ex.

$$P(E \cup F) = P(E) + P(F)$$
$$= \frac{4}{52} + \frac{12}{52} = \boxed{\frac{16}{52}}$$

Let E: Face card
F: ~~a~~ a black card.   Non-mutually exc.

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$
$$\frac{12}{52} + \frac{26}{52} - \frac{6}{52} = \frac{32}{52}$$

Independent vs Dependent Events.

(and)
Independent Events - Two or more events that have no affect on the outcome of eachother.

If E and F are independent events in the sample space S. Then...

$$P(\overset{\text{"and"}}{E \cap F}) = P(E)P(F)$$

Ex. Find the probability of rolling a $\overset{E}{3}$ and $\overset{F}{4}$ and $\overset{G}{5}$ after 3 rolls.

$$P(E \cap F \cap G) = P(E)P(F)P(G) = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)$$

$$= \frac{1}{6^3} = \boxed{\frac{1}{216}}$$

"Without replacement" $\Leftarrow$ not independent

Let E and F be dependent events in the sample space S.

Then...

"given the condition that..."

$$P(E \overset{\text{"and"}}{\cap} F) = P(E)\,P(F|E)$$

Ex. From an urn containing 15 blue balls and 10 red balls, six balls are drawn at random. What is the P(having at least 1 red ball)?

$$P(E') = P(\text{having 6 blue balls}) = \left(\frac{15}{25}\right)\left(\frac{14}{24}\right)\left(\frac{13}{23}\right)\left(\frac{12}{22}\right)\left(\frac{11}{21}\right)\left(\frac{10}{20}\right)$$

Compound Events

"or"
$\cup$ – union

$\cap$ – Intersection
"and"

Mutually Exclusive Events. "or"

Two or more events that do not share any outcomes.

If E and F are mutually exclusive events in the sample space S, then...

$$\underset{\text{"or"}}{P(E \cup F)} = P(E) + P(F)$$

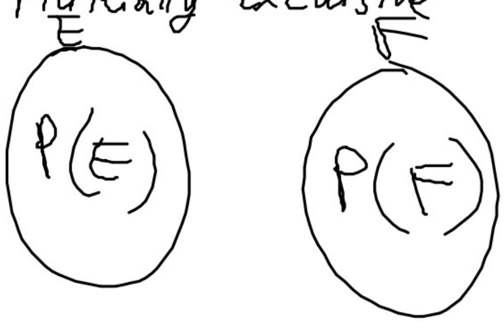
$$P(E_1 \cup E_2 \cup E_3 \cup \dots) = P(E_1) + P(E_2) + \dots$$

Non-Mutually Exclusive Events

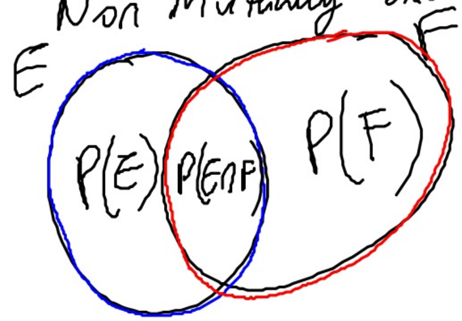
If E and F are non-mutually exclusive events in the sample space S, then...

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

Mutually Exclusive



Non Mutually Exclusive



Ex. Let E: drawing a Queen
F: drawing an Ace   Mutually Exclusive.

$$P(E \cup F) = P(E) + P(F)$$
$$= \frac{4}{52} + \frac{4}{52} = \boxed{\frac{8}{52}}.$$

Let E: drawing a queen   Non-mutually Exclusive.
F: drawing a black card.

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$
$$\frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \boxed{\frac{28}{52}}.$$

Independent vs Dependent Events "and".

Independent Events – Events that do not affect the outcome of each other.

$$P(E \cap F) = P(E)P(F)$$

Ex. Rolling a die twice, what is the probability of rolling a 5 and a 6?

E (the 5), F (the 6)

Ind.

$$P(E \cap F) = P(E)P(F) = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \boxed{\frac{1}{36}}$$

Ex. E: drawing a Face card
F: drawing a 7

"w/o replacement!!"

Not independent

$$P(E\cap F)=P(E)P(F|E)=\left(\frac{12}{52}\right)\left(\frac{4}{51}\right)$$

$$=\boxed{\frac{48}{2652}}$$

Non-Independent Events

$$P(E\cap F)=P(E)P(F|E)$$

"given that..."

Ex. From an urn containing 15 blue balls and 10 red balls Six balls are chosen. What is the probability that at least one is red? w/o replacement.
(E)

$P(E') = P(\text{all six are blue}) = P(B \cap B \cap B \cap B \cap B \cap B)$

Not independent

$1 - P(E') = P(E)$

$1 - P(E') = \left(\frac{15}{25}\right)\left(\frac{14}{24}\right)\left(\frac{13}{23}\right)\left(\frac{12}{22}\right)\left(\frac{11}{21}\right)\left(\frac{1\cdot}{2\cdot}\right)$

$\approx \boxed{97\%}$

Ex. E: Drawing a face card   w/o replacement.

F: Drawing a diamond

Not independent.

$$P(E\cap F) = \left(\frac{3}{52}\right)\left(\frac{12}{51}\right) \cup \left(\frac{9}{52}\right)\left(\frac{13}{51}\right)$$

$$= \frac{36}{2652} + \frac{117}{2652} = \boxed{\frac{152}{2652}}$$