

Probability

$$\frac{\text{Part}}{\text{Whole}}$$

Experiment — A process that gives definite results.

Outcome — A definite result of an experiment.

ex. Experiment: tossing a coin
Outcome: lands heads up.

Sample Space (S) The set of all outcomes.

(whole) Ex. Tossing a coin.

Sample space: $\{H, T\}$

Ex. rolling a die

Sample space: $\{1, 2, 3, 4, 5, 6\}$

Ex. Lottery drawings.

Event - If S is the sample space, then an event is any subset of the sample space.

Ex. Tossing a coin

Event: tossing a "heads"

$S = \{H, T\}$

Ex. Rolling a die

Event: Rolling an odd #
(Part)

$S = \{1, 2, 3, 4, 5, 6\}$

Ex. In tossing a coin 3 times calculate the probability of getting $S = \{HHH, HHT, HTH, HTT, THH, TTH, THT, TTT\}$

a) Exactly two heads

$$\frac{3}{8}$$

b) At least two heads.

$$\frac{4}{8}$$

c) No heads

$$\frac{1}{8}$$

d) All heads

$$\frac{1}{8}$$

~~#~~ Ex. A five-card poker hand is drawn from the standard 52-card deck. What is the probability that all of them are Spades?

$$S = 52C_5 = 2,598,960$$

$$E = 13C_5 = 1287$$

$$\Rightarrow \frac{1287}{2,598,960} \approx \boxed{0.0005}$$

Complement of an event. - Let S be the sample space of an experiment and E an event. Then,

$$P(E') = 1 - P(E) \text{ is called the complement.}$$

Ex. An urn contains 10 red balls and 15 blue balls. 6 balls are drawn at random. What is the probability that at least one is red?

$$| \quad \frac{b}{1} \quad \frac{b}{2} \quad \frac{b}{3} \quad \frac{b}{4} \quad \frac{b}{5} \quad \frac{b}{6}$$

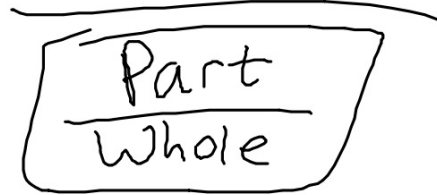
$$S = {}^{25}C_6 = 177,100 \quad \left| \quad \frac{5005}{177,100} \approx \boxed{97\%} \right.$$
$$E = {}^{15}C_6 = 5005$$

#1-8

6) 110 people at a meeting. They each shake hands w/ everyone else. How many handshakes are there?

$$110C_2 = \boxed{5995}$$

Probability



Experiment - A process that yields definite results.

Outcome - A definite result of an experiment.

Sample Space - The set of all outcomes in an experiment.

> ex. Experiment: Tossing a coin

outcome: getting a "heads"

Sample Space: $\{H, T\}$.

Event. - If S is the sample space, then an event is any subset of the sample space.

Ex. Rolling a die

$$S = \{1, 2, 3, 4, 5, 6\}$$

Event: Rolling an even #.

$$= \left\{ \frac{2}{6} \right\}$$

Ex. Lottery drawing Δ

Ex. In tossing a coin 3 times, what is the probability of getting ... $S = \{HHH, HHT, HTH, HTT, THT, TTH, TTT\}$.

a) Exactly two heads
HHT, HTH, THT
 $\frac{3}{8}$

b) at least two heads.
HHH
 $\frac{4}{8}$

c) No heads
TTT
 $\frac{1}{8}$

d) all heads
HHH
 $\frac{1}{8}$

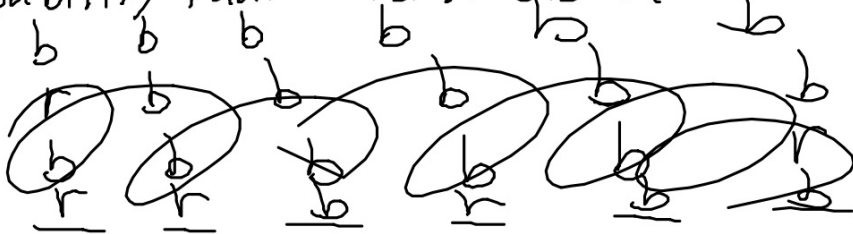
Ex A five-card hand is drawn at random from the standard 52-card deck. What is the probability that all 5 cards are spades?

$$S = 52 C_5 = 2,598,960 \quad \Rightarrow \quad \frac{1287}{2,598,960} \approx 0.0005$$
$$E = 13 C_5 = 1287 \quad \approx \boxed{0.05\%}$$

Complement of an event - Let S be a sample space of an ~~experiment~~ experiment and E be an event. Then,

$1 - P(E) = P(E')$ is called the complement probability.

Ex: An urn contains 10 red balls and 15 blue balls.
Six balls are drawn at random. What is the probability that at least one of them is red?



$$1 - P(\text{all blue})$$

$${}_{25}C_6 = 177,100 \leftarrow S$$

$${}_{15}C_6 = 5005 \leftarrow E$$

$$1 - \frac{5005}{177,100}$$

$$\approx .972$$

$$\approx 97\%$$

Probability

$\frac{\text{Part}}{\text{Whole}}$

Experiment - A process that yields definite results

Outcome - A definite result of an experiment.

Ex. Experiment: tossing a coin

Outcome: getting a "heads"

Experiment: rolling a die

Outcome: rolling a "3"

Sample space (S) — The set of all outcomes.

(Whole)

Ex. Tossing a coin

$$S = \{H, T\}$$

Ex. Rolling a die

$$S = \{1, 2, 3, 4, 5, 6\}$$

Event (E) — If S is the sample space, then an event (part) is any subset of the sample space.

$$\text{Probability} = \frac{n(E)}{n(S)} = \frac{\text{part}}{\text{whole}}$$

Ex. Tossing a coin

Event: Heads = $\frac{1}{2}$

Ex. rolling a die.

Event: rolling an odd #
 $S = \{1, 2, 3, 4, 5, 6\} \Rightarrow \frac{3}{6}$

Ex. In tossing a coin thrice, what is the probability
of getting - $S = \{HHH, HHT, HTH, HTT, THT, THT, TTH, TTT\}$

a) Exactly two heads,
HHT, HTH, THH

$$\frac{3}{8}$$

b) at least two heads,
HHH

$$\frac{4}{8}$$

c) No heads
TTT

$$\frac{1}{8}$$

d) All heads
HHH

$$\frac{1}{8}$$

Ex. Winnings
the lottery

Ex. A five-card hand is drawn at random. What is the probability that all 5 cards are spades?
(Note: Standard deck has 52 cards)

$$S = 52C_5 = 2,598,960$$

$$E = 13C_5 = 1287$$

$$\Rightarrow P(E) = \frac{1287}{2,598,960}$$

$$\approx 0.0005$$

Complement of an event

$$\approx 0.05\%$$

Let S be the sample space, and E be the event. Then,

$1 - P(E) = P(E^c)$ is called the complement.

Ex An urn contains 10 red balls and 15 blue balls. Six balls are drawn at random. What is the probability that at least one of them is red?

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$$S = {}_{25}C_6 = 177,100$$

$$E' = {}_{15}C_6 = 5005$$

$$\Rightarrow P(E) = 1 - P(E')$$

$$= 1 - \frac{5005}{177100}$$

$$\approx 97.2\%$$

#1-8