

Permutations & Combinations

Ex a) How many ways can you order letters: ABCDWXYZ?

$$\frac{8 \times 7 \times 6 \times 5 \times 4 \times \boxed{3 \times 2 \times 1}}{3!} = \boxed{40320}$$
$$= \boxed{8!}$$

b) How many ways can you order 5 of the above letters?

$$8 \times 7 \times 6 \times 5 \times 4 = \boxed{6720}$$
$$= \frac{8!}{(8-5)!}$$

Permutation - From a set of distinct objects,
Permutation is the ordering of those objects.

$$\text{i.e. } P(n, r) = nPr = \frac{n!}{(n-r)!}$$

From Previous Ex. Letters: ABCDXYZ
take 5 letters to order.

$$P(8, 5) = {}_8P_5 = \frac{8!}{(8-5)!} = \frac{8!}{3!} = \boxed{6720}$$

Ex. A movie theater has 7 movie posters with 4 slots on their wall to hang them. How many ways can the theater arrange the posters?

$$P(7, 4) = 7P_4 = \frac{7!}{(7-4)!} = \frac{7!}{3!} = 840$$

Ex. From the set of letters: ABCD WXYZ, in how many ways can 5 letters be chosen?

$$\begin{aligned} & \underline{8} \quad \underline{7} \quad \underline{6} \quad \underline{5} \quad \underline{4} \\ = & \frac{8!}{5!(8-5)!} = \frac{8!}{5!3!} = \boxed{56}. \end{aligned}$$

Combination - Choosing of subset of elements out of the whole set.

$$P(8,8) \quad \text{vs.} \quad C(8,8) = 1$$
$$= 8 \times 7 \times 6 \times \dots$$
$$= 8! = 40320$$
$$C(n,r) = nCr = \boxed{\frac{n!}{r!(n-r)!}}$$

Ex. Lee's pizza has 16 different toppings to choose from. How many different 3-topping pizzas can be made?

$$16C_3 = \frac{16!}{3!13!} = \boxed{560}$$

1, 7, 10, 15, 19, 28, 33, 40, 42, 43, 53, 60,
70.

Permutations and Combination,

Ex^{In} (a) How many different ways can you arrange the letters: ABCD WXYZ?

$$\begin{aligned} & \underline{8} \times \underline{7} \times \underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \\ & = 8! = \boxed{40320} \end{aligned}$$

b) In how many different ways can you arrange 5 of those letters above?

$$\begin{aligned} & \underline{8} \times \underline{7} \times \underline{6} \times \underline{5} \times \underline{4} = \frac{8!}{3!} = \frac{8!}{(8-5)!} \\ & = \boxed{6720} \end{aligned}$$

Permutation - The number of ways a certain subset within a set of elements can be ordered.

$$P(n, r) = nPr = \frac{n!}{(n-r)!}$$

set subset

From previous ex. 8 letters = n
5 letters out of 8 = r

$$P(8, 5) = 8P5 = \frac{8!}{(8-5)!} = 6720$$

key words: "order", "arrange"

ex. 5 runners compete in a race. How many top 3 finishes exist?

$$P(5, 3) = 5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \boxed{60}.$$

ex.2 How many different ^{lowercase} 3-lettered computer passwords exist?

$$P(26, 3) = 26P_3 = \frac{26!}{(26-3)!} = \frac{26!}{23!} = \boxed{15600}$$

ex. From the letters ABCD~~WX~~YZ, in how many ways can 5 letters be chosen?

$$\underline{8} \times \underline{7} \times \underline{6} \times \underline{5} \times \underline{4}$$

$$= \frac{8!}{5!(8-5)!} = \boxed{56}$$

Combination - The number of ways a subset of the set of elements can be chosen.

$$C(n, r) = nCr = \frac{n!}{r!(n-r)!}$$

$$P(8, 8) = {}_8P_8 = 8! = \boxed{40320}$$

$$C(8, 8) = {}_8C_8 = \boxed{1}$$

Ex. At Lee's pizzeria there are 17 different toppings to choose from. How many 5-topping pizza orders exist?

$$C(17, 5) = {}_{17}C_5 = 6182$$

Permutation & Combination

Ex) Using the letters ABCDWXYZ, in how many different ways can they be arranged?

$$\underline{8} \times \underline{7} \times \underline{6} \times \underline{5} \times \underline{4} \times \boxed{\underline{3} \times \underline{2} \times \underline{1}}$$
$$= 8! = 40320$$

3!

b) Using the same letters from above, in how many different ways can we arrange 5 of them?

$$\underline{8} \times \underline{7} \times \underline{6} \times \underline{5} \times \underline{4} = \frac{8!}{3!} = \frac{8!}{(8-5)!} = \boxed{6720}$$

Permutation - Arrangement or ordering of certain subset of elements out of the whole set of elements

$$\text{i.e. } P(n, r) = nPr = \frac{n!}{(n-r)!}$$

$$\text{in our previous ex. } n=8, r=5 \Rightarrow P(8, 5) = 8P_5 = \frac{8!}{(8-5)!}$$

ex. Out of 7 people, how many ways can you elect a president, vice-president, and a secretary?

$$P(7, 3) = 7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!} = \boxed{210}$$

P	UP	S
A	Sk	J
Sk	J	A

ex. Using the letters: ABCDXYZ, in how many ways can you select 5 letters?

$$8 \times 7 \times 6 \times 5 \times 4 = \frac{8!}{5!(8-5)!} = \frac{8!}{5!3!} = \boxed{56}$$

Combination - Choosing a certain subset of elements out of the whole.

ex. At Lee's Pizzeria there are 12 toppings to choose from. How many different 4-topping pizza orders can be made

$$\begin{array}{cccc} P & H & M & J \\ 3 & H & P & B \\ 5 & H & J & P \end{array} \quad C(12, 4) = 12C4 = \boxed{495}$$

