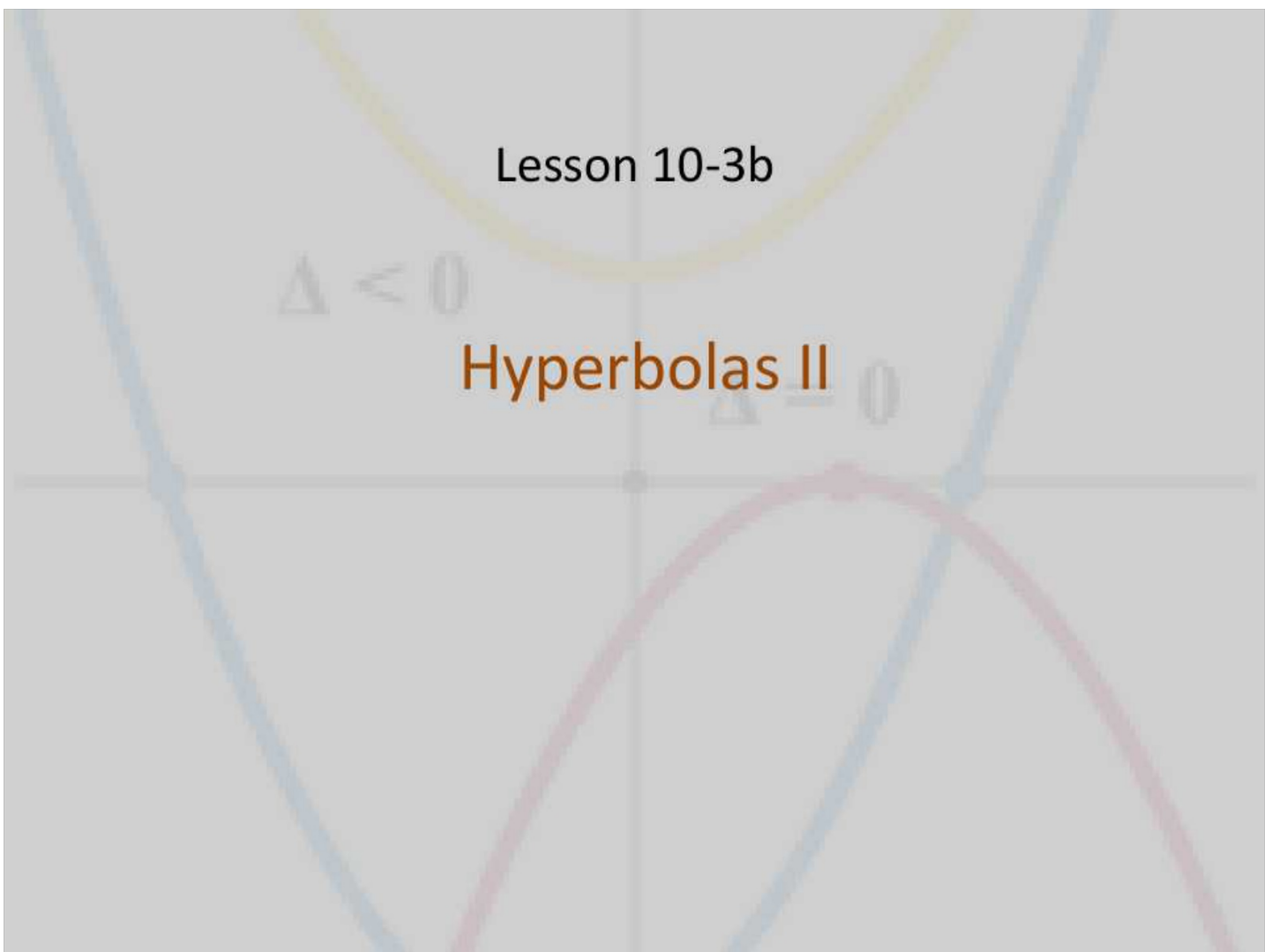


Lesson 10-3b

$\Delta < 0$

Hyperbolas II

$\Delta = 0$



Objective

Students will...

- Be able to derive the equation of standard hyperbolas, given the vertices and foci, and/or the asymptotes.

Equations and Graphs of Hyperbolas

Using the distance formula, we can see that parabolas have the following equations: **for $a > 0$ and $b > 0$ (not $a > b$)**

Horizontal

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

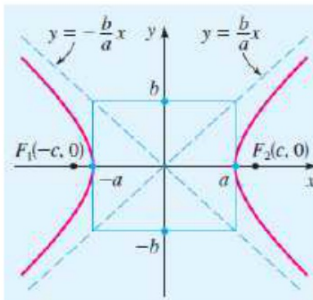
Vertices: $(\pm a, 0)$

Covertices: $(0, \pm b)$

Transverse Axis: Horizontal length $2a$

Asymptotes: $y = \pm \frac{b}{a}x$ $\frac{\Delta y}{\Delta x}$

Foci: $(\pm c, 0)$, $c^2 = a^2 + b^2$



Vertical

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

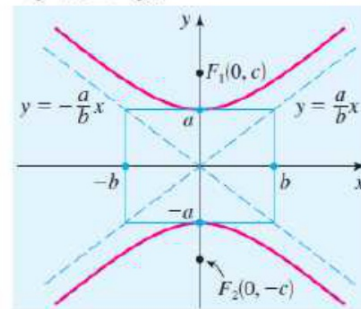
$(0, \pm a)$

$(\pm b, 0)$

Vertical length $2a$

$y = \pm \frac{a}{b}x$ $\frac{\Delta y}{\Delta x}$

$(0, \pm c)$, $c^2 = a^2 + b^2$

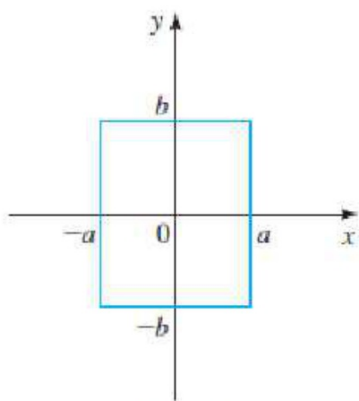


Sketching the Hyperbola

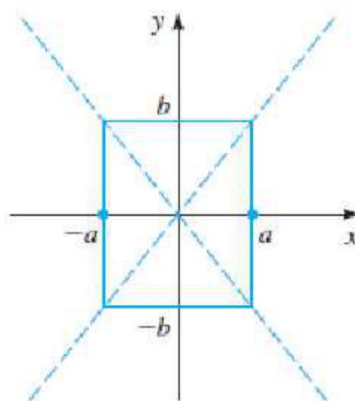
Here is a guidelines you can use to graph the hyperbola.

1. Sketch the **central box**, using the vertices and the covertices.
2. Sketch the **asymptotes**. These are the diagonals of the central box.
3. Plot the foci
4. Sketch the hyperbola.

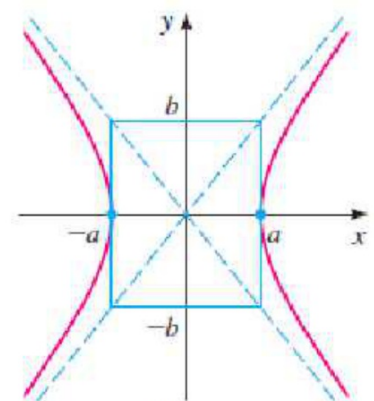
Ex.



(a) Central box



(b) Asymptotes

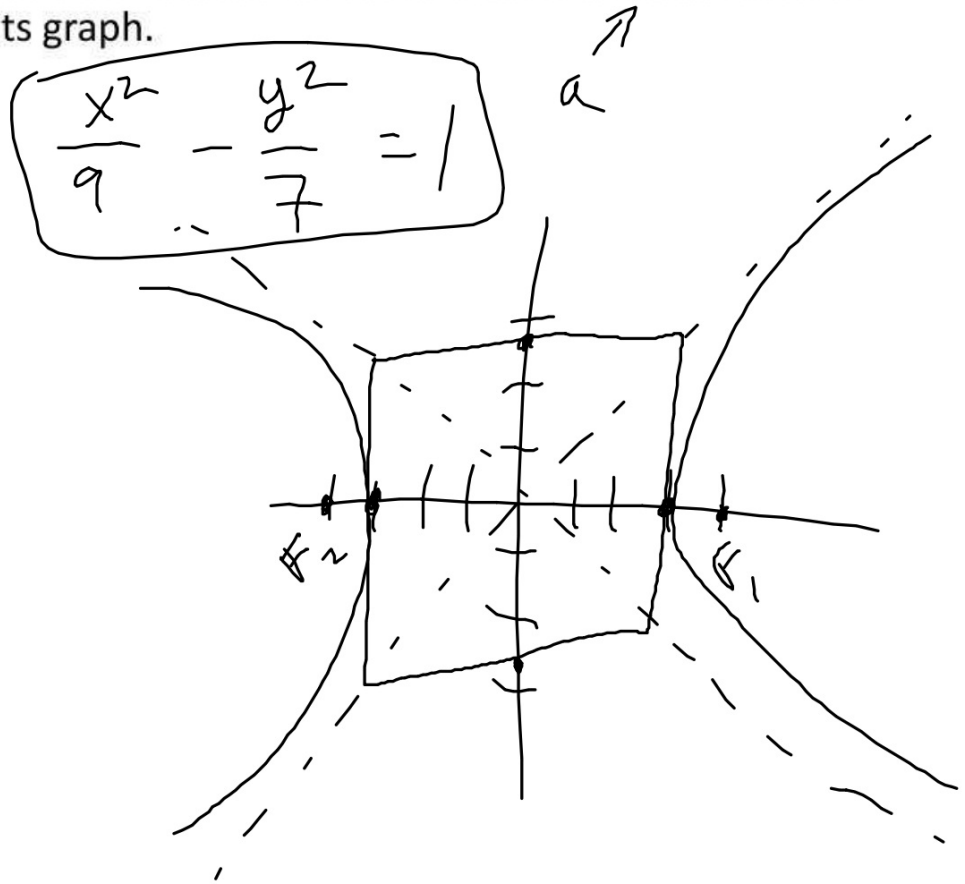


(c) Hyperbola

Example *horiz.*

Find the equation of the hyperbola with vertices $(\pm 3, 0)$ and foci $(\pm 4, 0)$. Sketch its graph.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 16 &= 9 + b^2 \\ 7 &= b^2 \\ \pm\sqrt{7} & \\ & b \end{aligned}$$



Example

Find the equation of the hyperbola with vertices $(\pm 2, 0)$ and foci $(\pm 3, 0)$. Sketch its graph.

$$\frac{x^2}{4} - \frac{y^2}{5} = 1$$

Example

Find the equation and the foci of the hyperbola with vertices $(0, \pm 2)$ and asymptotes $y = \pm 2x$. Sketch the graph.

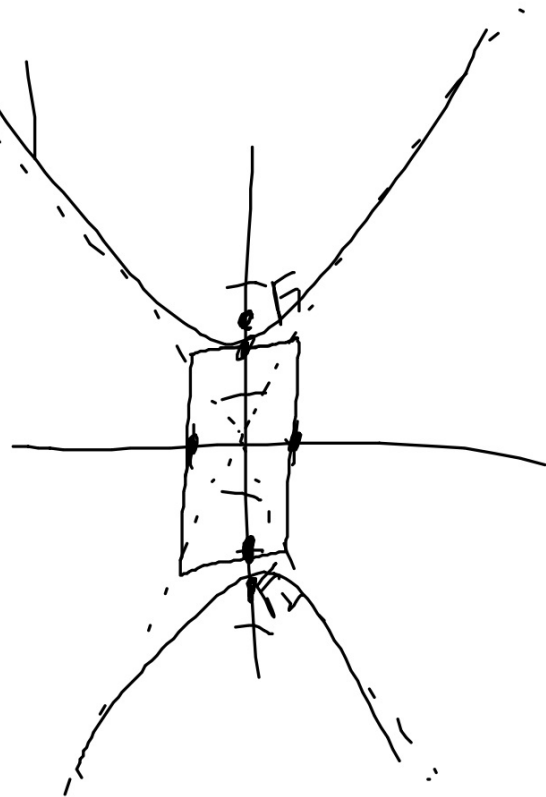
$$\frac{a}{b} = \frac{2}{1}$$

$$c^2 = 4 + 1$$

$$c^2 = 5$$

$$c = \pm\sqrt{5}$$

$$\frac{y^2}{4} - \frac{x^2}{1}$$



Example

Find the equation and the foci of the hyperbola with vertices $(0, \pm 4)$ and asymptotes $y = \pm 4x$. ~~Sketch the graph.~~

$$b = 4$$

$$\frac{y^2}{16} - \frac{x^2}{1} = 1$$

Example

Find the equation of the hyperbola with vertices $(0, \pm 6)$, given that it passes through the point $(5, -9)$

$$\frac{81}{36} - \frac{25}{b^2} = 1$$
$$\Rightarrow 4 \left(\frac{9}{4} - \frac{25}{b^2} \right) = 4$$
$$\Rightarrow \frac{9}{1} - \frac{100}{b^2} = 4 - 9 \Rightarrow -\frac{100}{b^2} = -5 \Rightarrow b^2 = \frac{100}{5} = 20$$

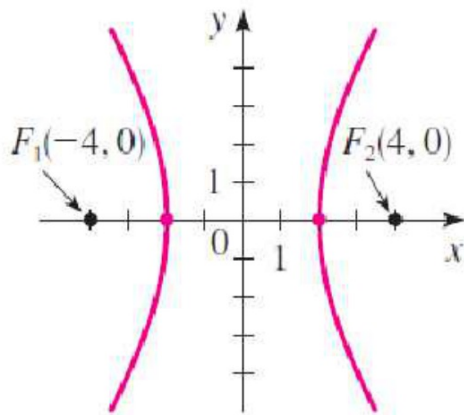
$$\frac{y^2}{36} - \frac{x^2}{20} = 1$$

$\swarrow a$

Homework Problem

Find the equation for the hyperbola whose graph is shown.

17.



Homework Problem

Find the equation for the hyperbola with the given conditions.

38. Foci $(0, \pm 1)$, length of the transverse axis $1. = 2a$

$$a = \frac{1}{2}.$$

Homework 5/29

TB pgs. 768-769 #17, 19, 21, 27, 29, 31, 32, 34, 37, 38