

## Objective

#### Students will...

- Be able to understand partial fraction decomposition.
- Be able to use partial fraction decomposition to integrate rational functions.

### **Partial Fractions**

Consider the following problem:

$$\int \frac{1}{x^2 - 5x + 6} dx$$

This problem can be done using Trig Substitution (Section 8.4), but it becomes extremely tedious. (See example on pg. 552). Using Trig Substitution we see that  $\int \frac{1}{x^2 - 5x + 6} dx = \iint_{x-2} \frac{1}{x-2} dx$   $\left| \int_{x-2} \left( x - y \right) \right| - \left| \int_{x-2} \left( x - y \right) \right| + \left( \int_{x-2} \left( x - y \right) dx$ 

ln ( ---

#### **Partial Fractions**

Now, suppose you were told that...

$$\frac{1}{x^2 - 5x + 6} = \frac{1}{x - 3} - \frac{1}{x - 2}$$

Then,
$$\int \frac{1}{x^{2}-5x+6} dx = \int \frac{1}{x^{2}} dx - \int \frac{1}{x^{2}} dx = \ln|x-3| - \ln|x-2| + C.$$

#### **Partial Fractions**

Partial Fraction technique allows us to "decompose" a fraction into a

sum of two "easier" fractions. You have learned how to combine fractions like 
$$\frac{(X+1)}{(X+1)} \frac{1}{x-2} - \frac{1}{x+3} = \frac{(X+1)}{(X+1)} \frac{1}{(X+1)} = \frac{1}{(X+1)} \frac{1}{(X+1)} =$$

This method of partial fraction simply reverses this process: 
$$\frac{5}{x^2 + x - 6} = \frac{5}{(x - 2)(x + 3)} = \frac{?}{x - 2} + \frac{?}{x + 3}$$

### **Guidelines for Partial Fractions**

#### DECOMPOSITION OF N(x)/D(x) INTO PARTIAL FRACTIONS

 Divide if improper: If N(x)/D(x) is an improper fraction (that is, if the degree of the numerator is greater than or equal to the degree of the denominator), divide the denominator into the numerator to obtain

$$\frac{N(x)}{D(x)}$$
 = (a polynomial) +  $\frac{N_1(x)}{D(x)}$ 

where the degree of  $N_1(x)$  is less than the degree of D(x). Then apply Steps 2, 3, and 4 to the proper rational expression  $N_1(x)/D(x)$ .

Factor denominator: Completely factor the denominator into factors of the form

$$(px+q)^m$$
 and  $(ax^2+bx+c)^n$ 

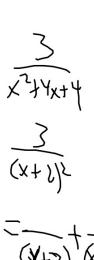
where  $ax^2 + bx + c$  is irreducible.

 Linear factors: For each factor of the form (px + q)<sup>m</sup>, the partial fraction decomposition must include the following sum of m fractions.

$$\frac{A_1}{(px+q)} + \frac{A_2}{(px+q)^2} + \cdots + \frac{A_m}{(px+q)^m}$$

**4. Quadratic factors:** For each factor of the form  $(ax^2 + bx + c)^n$ , the partial fraction decomposition must include the following sum of n fractions.

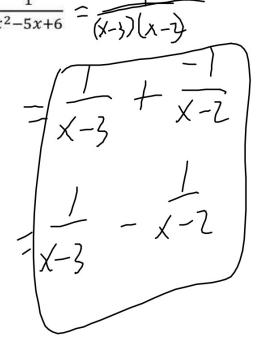
$$\frac{B_1x + C_1}{ax^2 + bx + c} + \frac{B_2x + C_2}{(ax^2 + bx + c)^2} + \cdots + \frac{B_nx + C_n}{(ax^2 + bx + c)^n}$$



# **Examples**

Write the partial fraction decomposition for 
$$\frac{1}{x^2-5x+6} = \frac{1}{(x-1)(x-1)}$$

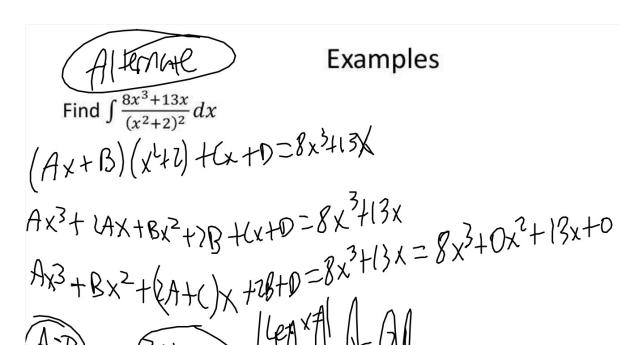
$$A(x-2) + B(x-3) = 1$$
Let  $x=2$ 
 $A(3-2) = 1$ 
 $B(2-3) = 1$ 
 $A=1$ 
 $B(2-3) = 1$ 
 $B(2-3) = 1$ 



$$\frac{q}{(x+1)^{2}} = \frac{q(x+1)^{-2}}{du_{1}} \frac{du_{2}}{du_{1}} \frac{du_{2}}{du_{2}} \frac{du_{3}}{du_{4}} \frac{du_{4}}{du_{5}} \frac{du_{5}}{du_{5}} \frac{du_$$

$$\frac{Cx+0}{x^{2}+4} = \frac{Cx}{x^{2}+4} + \frac{D}{x^{2}+4}$$
Examples  $du = 2x^{2} + \frac{1}{x^{2}} + \frac{2}{x^{2}+4} + \frac{2$ 

Find  $\int \frac{8x^3 + 13x}{(x^2 + 2)^2} dx$   $\frac{1}{2} \frac{1}{2} \frac{1}{$  $(Ax+B)(x^{2}+2) + (x+0) = 8x^{3}+13x$   $(Ax+B)(x^{2}+2) + (x+0) = 8x^{3}+13x$  (b+x=-1) (b+x=-1)



# Homework 3/23

8.5 #1-6, 7-31 (odd)