Lesson 10-2b

Ellipses: Eccentricity

Objective

Students will…

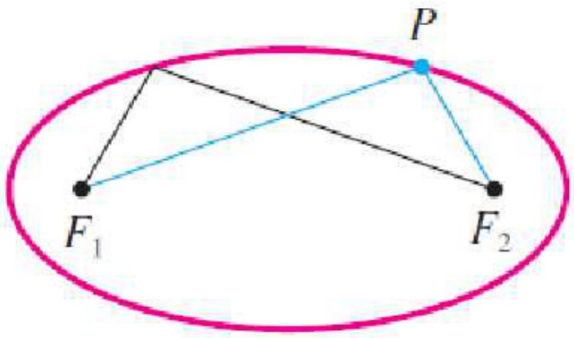
- Be able to know and set up the ratio of eccentricity for any given ellipse.
- Be able to derive equation of an ellipse given the eccentricity and the foci.

Ellipse

Here, we want to geometrically define what an ellipse is.

<u>Geometric Definition of an Ellipse</u>- An ellipse is the set of all points in the plane the sum whose distances from two fixed points F_1 and F_2 is a constant. These two fixed points are **foci** (plural of focus) of the ellipse.

Ex.



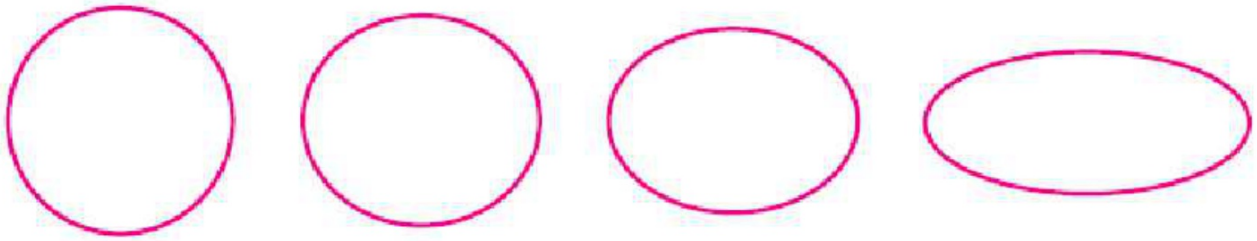
Eccentricity

We know that an ellipse is an oval shape, which can be considered as a **stretched circle**. A numerical value that indicates how close an ellipse is to a circular shape is known as the **eccentricity** of a circle.

Eccentricity- Given a standard ellipse with the equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ or $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, the eccentricity $\boldsymbol{e}$ is the number $e=\frac{c}{a}$, where $c^2=a^2-b^2$ and for every ellipse, $0<e<1$.

Eccentricity of an Ellipse

With that being said, eccentricity is useful in telling us just how close an ellipse looks like a circle. If e is closer to 0, then the ellipse looks more like a circle. If e is closer to 1, then the ellipse looks more like an oval.



$e = 0.1$ $e = 0.5$ $e = 0.68$ $e = 0.86$

Example

Find the eccentricity of the following ellipse.

$$y = \frac{x^2}{25} + \frac{y^2}{9} = 1$$

$a^2 = 25$

$a = 5$

$c^2 = a^2 - b^2$

$= 25 - 9$

$= 16$

$c = \pm 4$

$e = \frac{4}{5} = 0.8$

Example

Find the eccentricity of the following ellipse.

$$y = \frac{x^2}{16} + \frac{y^2}{25} = 1$$

$c = \pm 3$.

a^2 $a = 5$

$e = \frac{3}{5} = 0.6$.

Example

Find the equation of the ellipse with foci $(0, \pm 8)$ and eccentricity $e = \frac{4}{5}$.

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$c^2 = a^2 - b^2$

$64 = 100 - b^2$

$b^2 = 36$

$\frac{c}{a} = \frac{8}{10}$

$\Rightarrow \boxed{\frac{x^2}{36} + \frac{y^2}{100} = 1}$

Example

Find the equation of the ellipse with foci $(0, \pm 20)$ and eccentricity $e = \frac{4}{5}$.

$400 = 625 - b^2$

$225 = b^2$

$\frac{x^2}{225} + \frac{y^2}{625} = 1$

Homework 5/21

TB pg. 759-760 #9-15 (odd) (just find the eccentricity!), 38-40

$$\frac{2x^2}{3} + \frac{y^2}{3} = \frac{3}{3}$$

$$-\frac{x^2}{3/2} + \frac{y^2}{3} = 1$$