Warm Up 4/21

Solve the following system of equations by using substitution.

1. $\begin{cases} 2x - y = 5 \\ x + 4y = 7 \end{cases}$

$-4y \quad -4y$

$x = 7 - 4y$

$2(7-4y) - y = 5$

$14 - 8y - y = 5$

$-9y = -9$

$y = 1$

$(3, 1)$

$2x - 1 = 5$

$x = 3$

Solve the following system of equations by using elimination.

(Hint: Eliminate the x)

2. $\begin{cases} 5(3x^2 + 2y) = (26)5 \\ 3(5x^2 + 7y) = (3)3 \end{cases}$

$\Rightarrow$

$\cancel{15x^2} + 10y = 130$

$- \quad \cancel{15x^2} + 21y = 9$

$-11y = 121$

$y = -11$

$+ \; - \; = \; -$

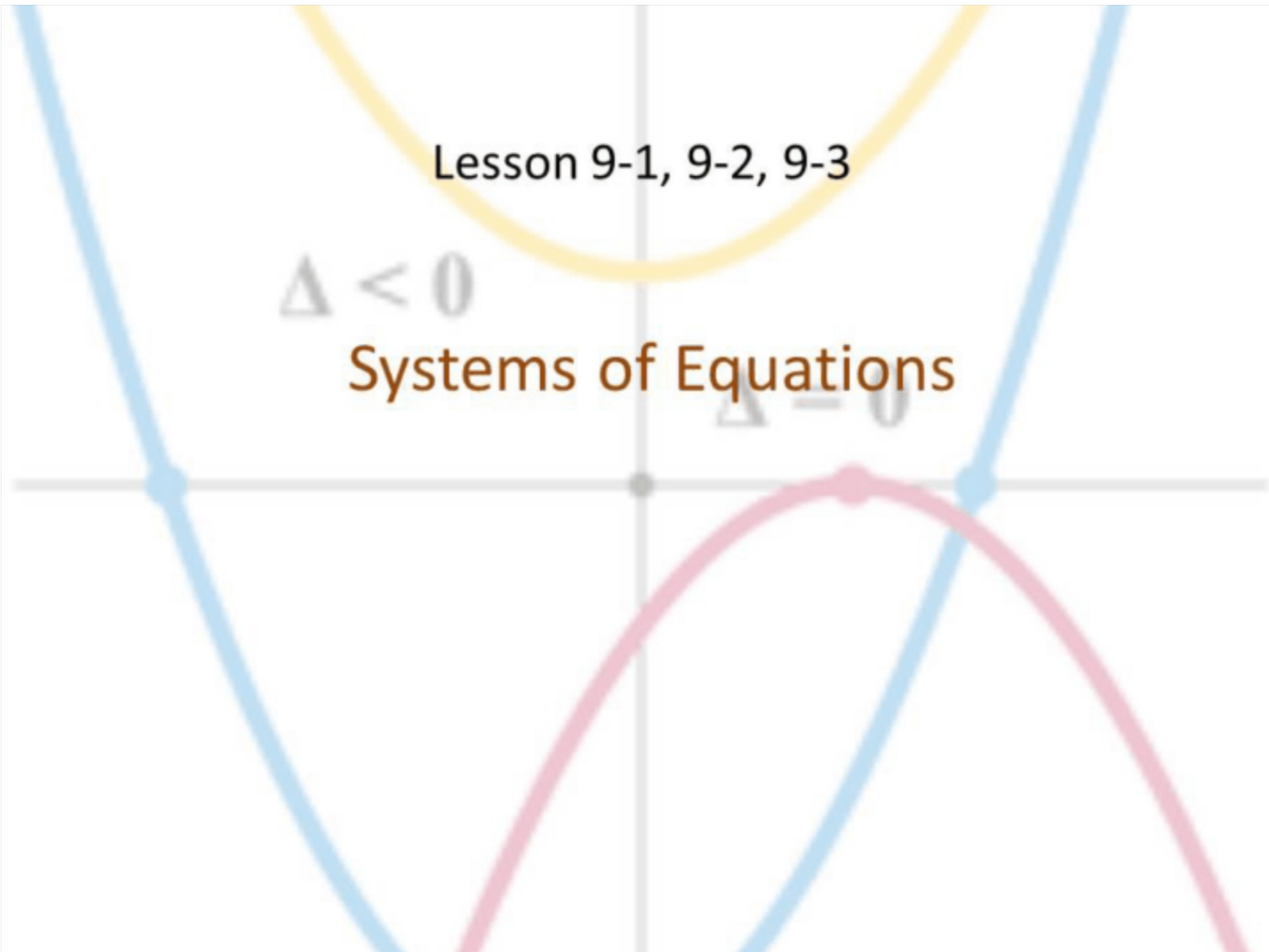
$3x^2 + 2(-11) = 2$

$3x^2 - 22 = 2$

$3x^2 = 4$

$\sqrt{x^2} = \sqrt{1}$

$x = \pm$


Lesson 9-1, 9-2, 9-3
Δ < 0
Systems of Equations

Objective

Students will…

- Be able to solve systems of equations using substitution or elimination.
- Be able to solve systems of multi-variable (more than 2) equations.

System of Equations

$3x - 5 = 10$

$4x - 3y = 3$

We all have solved a single equation before with one variable. Unfortunately, most application problems in the real-world often involve multiple variables that require more equations. A **system of equations** is simply a set of multiple equations in the same problem.

It turns out, the number of equations needed is dependent upon how many variables there are. Thus, for a two variables, we would need at least two equations. For three, at least three, and so on.

Ex. $\begin{Bmatrix} 4x - y = 8 \\ 2x - 8y = -12 \end{Bmatrix}$

$$\begin{Bmatrix} 3x + 7y + 9z = 8 \\ x - y + 2z = -911 \\ z = 88 \end{Bmatrix}$$

Solving Systems of Equations: Substitution

Geometrically, solutions to systems of equations are simply found where the graph of each equation meet. This is often tedious and uneasy to do, so the algebraic way can be quite useful. First method of solving system of equations is called **substitution**.

Substitution method calls for doing just that: substitute. Our goal is to isolate one of the variables, or solve for one of the variables in terms of the other, and **substitute** it into the other equations.

Ex. $\begin{cases} 2x - y = 5 \\ x + 4y = 7 \end{cases}$

$$\begin{cases} x - 2y - z = 1 \\ y + 2z = 5 \\ z = 3 \end{cases}$$

Solving Systems of Equations: Elimination

Another algebraic way of solving systems of equations would be using the method of **elimination**. The idea here is to eliminate one of the variables by either adding or subtracting the equations, in order to solve for one variable at a time. Sometimes we would have to first multiply one or more of those equations in order to make the coefficient numbers to match.

Ex. $\begin{Bmatrix} 3x^2 + 2y = 26 \\ 5x^2 + 7y = 3 \end{Bmatrix}$

$$+\begin{Bmatrix} 3x + \cancel{2y} = 14 \\ x - \cancel{2y} = 2 \end{Bmatrix}$$

$4x = 16$

$x = 4$

$y = 1$

Systems of multi-variable Equations:

$1+2+3+4+5+\cdots$

Gaussian Elimination

For system of **multi-variable** (more than 2) equations can be a bit more complicated. Based on the equations given, we may be able to use substitution to solve (as we did earlier). In many cases, however, we may not be able to use substitution. For these cases, we have to resort to using the Gaussian Elimination method, named after the mathematician Carl Frederich Gauss, who popularized it!

$\sum_{i=1}^{50} \cdots$

The idea hasn't changed here. The goal is to **solve one variable at a time** by way of eliminating all other variables. We **pick one of the equations** and use it to eliminate the same variable on the other equations.

Systems of multi-variable Equations: Gaussian Elimination

Example:

$$\begin{cases} x - 2y + 3z = 1 \\ x + 2y - z = 13 \\ 3x + 2y - 5z = 3 \end{cases}$$

$$\begin{array}{r} 3x + \cancel{2y} - 5z = 3 \\ + \; x - \cancel{2y} + 3z = 1 \\ \hline 4x - 2z = 4 \end{array}$$

$$\begin{array}{r} 3x + \cancel{2y} - 5z = 3 \\ - \; x + \cancel{2y} - z = 13 \\ \hline 2x - 4z = -10 \end{array}$$

$2(4x - 2z) = (4)2$

$2x - 4z = -10 \Rightarrow$

$$\begin{array}{r} 8x - \cancel{4z} = 8 \\ - \; 2x - \cancel{4z} = -10 \\ \hline 6x = 18 \end{array}$$

$x = 3$

$6 - 4z = -10$

$z = 4$

$3 + 2y - 4 = 13$

$y = 7$

$(3, 7, 4)$

~~$(3, 4, 7)$~~

You try…

Example:

$$
\begin{cases}
x - 2y + 3z = 1 \\
\cancel{2x} + \cancel{4y} - 2z = 26 \\
\cancel{2x} + \cancel{4y} - 7z = 11
\end{cases}
$$

$\ominus$

$5z = 15$

$z = 3$

$x - 2(6) + 9 = 1$

$x - 12 + 9 = 1$

$x = 4$

$\cancel{x} - 2y + 9 = 1$

$\ominus\ \cancel{x} + 2y - 3 = 13$

$-4y + 12 = -1$

$-4y = -24$

$y = 6$

No Solution and Infinitely Many Solutions

When solving systems of equations, we may end up with a system with no solution or a system with infinitely many solutions.

Example:

$$3\begin{cases} (8x - 2y) = (5)\,3 \\ 2(-12x + 3y) = (7)\,2 \end{cases}$$

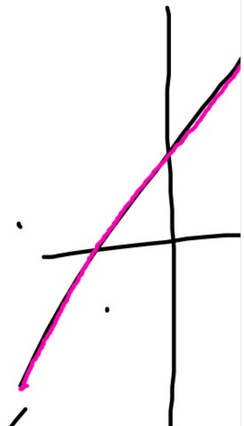
$$\begin{array}{r} \cancel{24x} - \cancel{6y} = 15 \\ -\cancel{24x} + \cancel{6y} = 14 \\ \hline 0 \neq 29 \end{array}$$

No sol.

$$4\begin{cases} (3x - 6y) = (12)\,4 \\ 3(4x - 8y) = (16)\,3 \end{cases}$$

$$\begin{array}{r} \cancel{12x} - \cancel{24y} = 48 \\ -\ \cancel{12x} - \cancel{24y} = 48 \\ \hline 0 = 0 \end{array}$$

Inf. solutions



Homework 4/21

Systems of Equations WKSHT