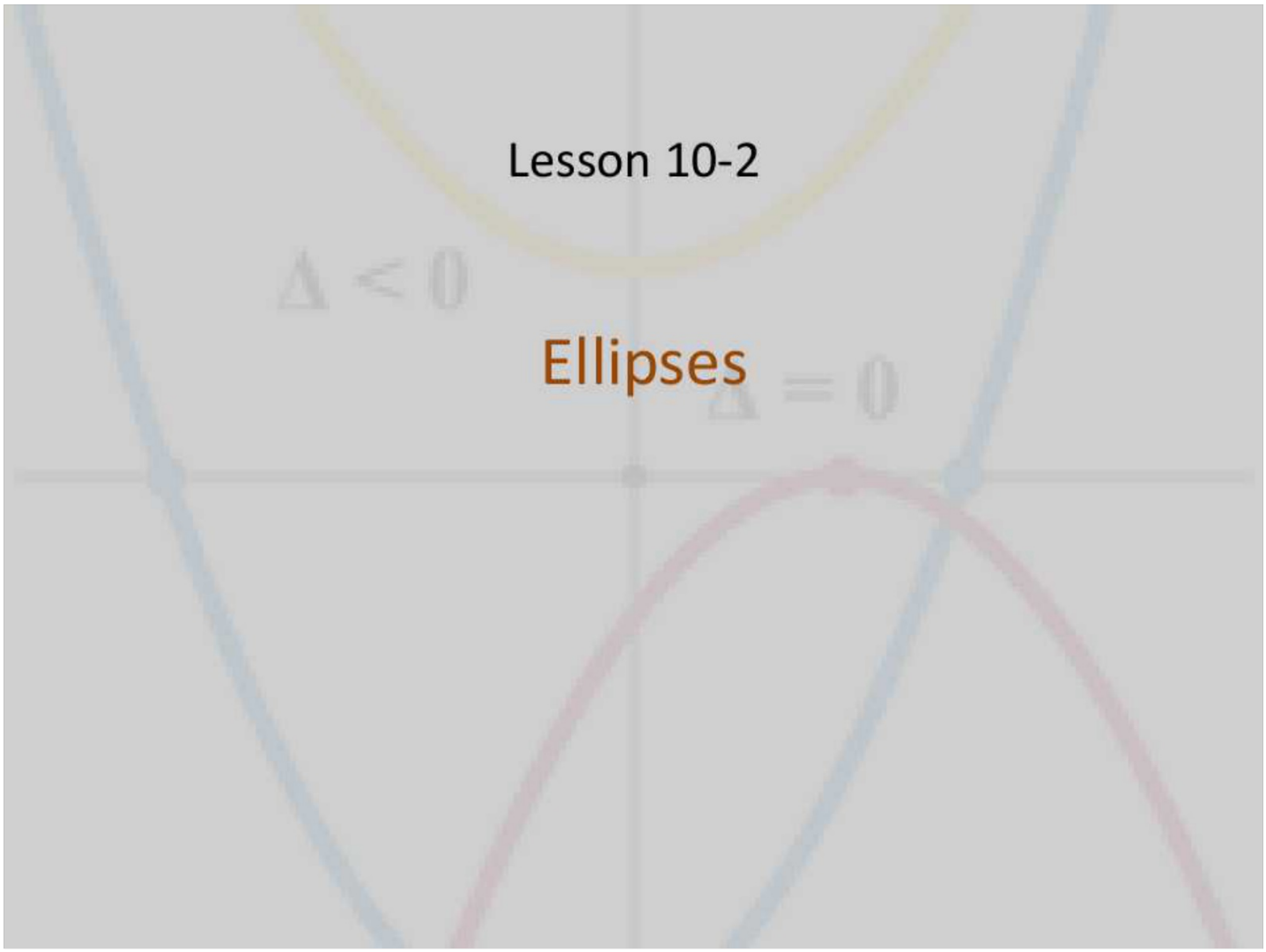
Lesson 10-2

Ellipses



Objective

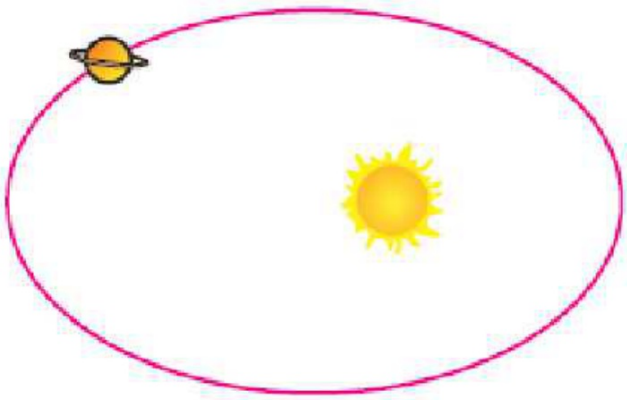
Students will…

- Be able to give a geometric definition of an ellipse.
- Be able to know the standard equation of ellipses.

Ellipse within a Cone

As seen from yesterday's video, a parabola can be cut out from a cone. Parabolas are easily found in the real-world.

Ellipse

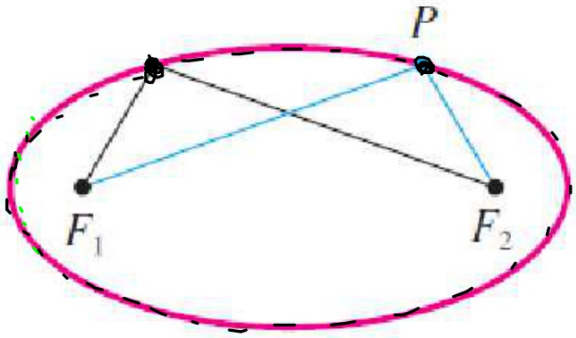


Ellipse

Here, we want to geometrically define what an ellipse is.

Geometric Definition of an Ellipse- An ellipse is the set of all points in the plane the sum whose distances from two fixed points F_1 and F_2 is a constant. These two fixed points are **foci** (plural of focus) of the ellipse.

Ex.



Equations and Graphs of Ellipses

Using the distance formula, we can see that parabolas have the following equations: ***for $a > b > 0$,***

Horizontal

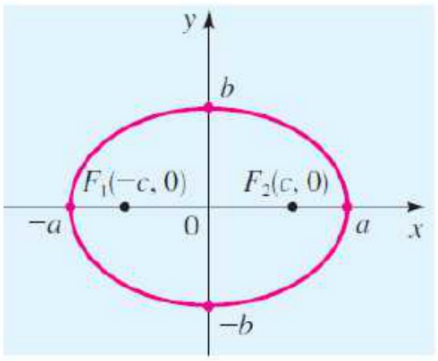
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Vertices: $(\pm a, 0)$ co-vertices: $(0, \pm b)$

Major Axis: Horizontal length 2a

Minor Axis: Vertical length $2b$

Foci: $(\pm c, 0)$, $c^2 = a^2 - b^2$



Vertical

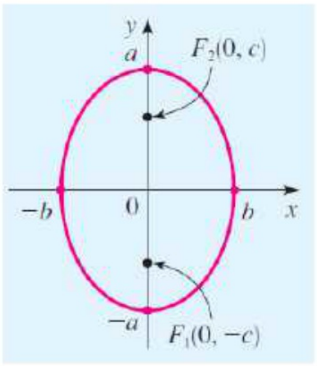
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$(0, \pm a)$ co-vert: $(\pm b, 0)$

Vertical length 2a

Horizontal length $2b$

$(0, \pm c)$, $c^2 = a^2 - b^2$



Example

horiz.

An ellipse has the equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$

a^2 b^2

Find the foci, vertices, and the lengths of the major and minor axes, and sketch the graph.

Vertices: $(\pm a, 0) = (\pm 3, 0)$

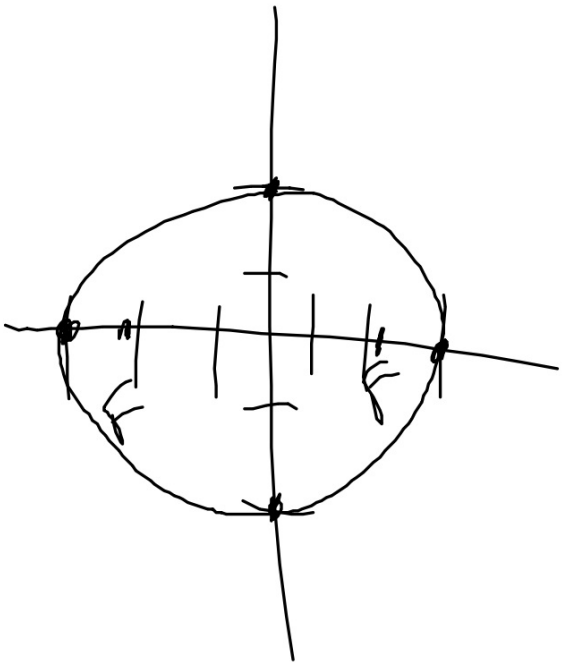
Co-Vert: $(0, \pm b) = (0, \pm 2)$

maj: 6 min: 4

foci: $(\pm c, 0) = (\pm\sqrt{5}, 0)$

$c^2 = a^2 - b^2$

$c^2 = 9 - 4 = \sqrt{5}$



Example

An ellipse has the equation $\frac{x^2}{36} + \frac{y^2}{25} = 1$

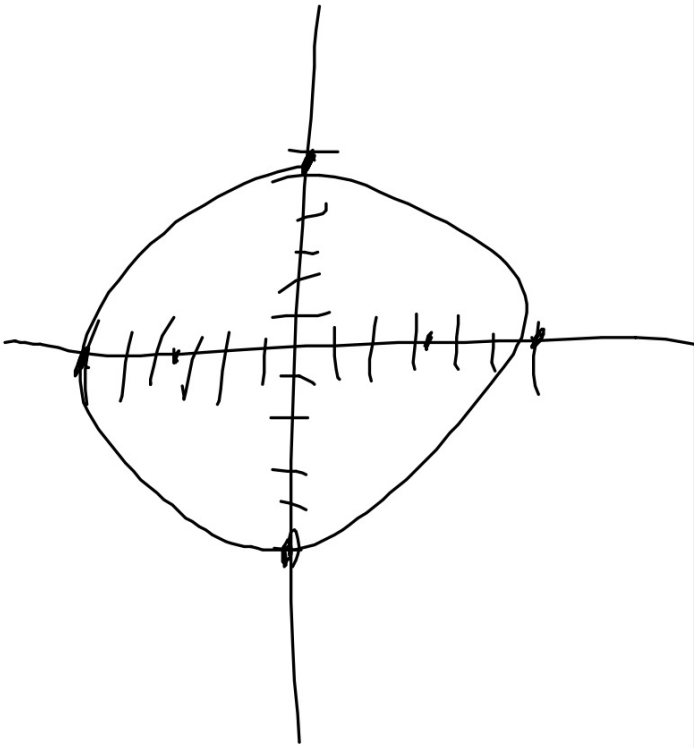
Find the foci, vertices, and the lengths of the major and minor axes, and sketch the graph.

Vert: $(\pm 6, 0)$

Co-vert: $(0, \pm 5)$

maj: 12 min: 10

foci: $(\pm\sqrt{11}, 0)$



Example

Find the foci of the ellipse $16x^2 + 9y^2 = 144$, and sketch its graph.

$\frac{16x^2}{144} + \frac{9y^2}{144} = \frac{144}{144}$

$\Rightarrow \frac{x^2}{9} + \frac{y^2}{16} = 1$

$b^2 \rightarrow 9$

$a^2 \rightarrow 16$

Example

The vertices of an ellipse are $(\pm 4, 0)$ and foci are $(\pm 2, 0)$. Find its equation and sketch the graph.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$c^2 = a^2 - b^2$$

Homework 5/19

TB pg. 759 #1-8