Lesson 8-2

Integration By Parts

Objective

Students will…

- Be able to use Integration By Parts technique to integrate.

Anti-Derivatives

You have learned by now (hopefully) that integrals "reverse" derivatives (hence the term anti-derivative). So for every differentiation technique (power rule, product/quotient rule, chain rule, etc.), it would make sense to have corresponding integration techniques.

We've learned the "anti-power rule," and U-Substitution (the "anti-Chain rule"). What about for derivatives that are formed as a result of a product rule? For this we have the **Integration By Parts Technique**.

Integration By Parts

Integration By Parts- If u and v are functions of x and have continuous derivatives, then $\int u\,dv = uv - \int v\,du$.

Proof: (Consider the Product Rule for Derivatives)

$$\frac{d}{dx}\underset{f\ g}{uv} = u'v + v'u = v\,du + u\,dv$$

$$\int \frac{d}{dx} uv = \int v\,du + \int u\,dv \Rightarrow \boxed{\int u\,dv = uv - \int v\,du.}$$

Guidelines for Integration By Parts

So, instead of choosing a u as done for U-Sub, here we need to choose or designate a function to be u and the other to be dv. For the u function you would need to find du by finding its derivative. For the dv function, you would need to find v by doing the anti-derivative.

Tips:

1. Try letting dv be the most complicated integrand.
2. Try letting u be the function with a simpler derivative. In other words, du is simpler than u.

This technique is mainly used when the integrand involves a product of algebraic and transcendental (e^x, $\ln x$, etc.) functions.

Ex. $\int x \ln x \, dx$ $\qquad$ $\int x^2 e^x dx$ $\qquad$ $\int e^x \sin x \, dx$

Examples

a. $\int \overset{u\,dv}{x e^x}\, dx = uv - \int v\, du$

$u = x \quad du = 1\, dx$

$dv = e^x dx \quad v = e^x$

$\int x e^x\, dx = x e^x - \int e^x\, dx$

$= \boxed{x e^x - e^x}$

Examples

b. $\int x^2 \ln x \, dx$

$u = \ln x \quad du = \frac{1}{x} dx$

$dv = x^2 dx \quad v = \frac{1}{3}x^3$

$$\int x^2 \ln x \, dx = \frac{1}{3}x^3 \ln x - \int \left(\frac{1}{3}x^3\right)\left(\frac{1}{x}\right) dx$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 \, dx$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\left(\frac{1}{3}x^3\right)$$

$$= \boxed{\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3}$$

Examples

$\sin^{-1}(x)$

$u \quad dv$

c. $\int_0^1 (\arcsin x)(dx)$

$u = \arcsin x \qquad du = \frac{1}{\sqrt{1-x^2}}\,dx$

$dv = 1\,dx \qquad v = x$

$u = 1 - x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$= x\arcsin x - \int \frac{x}{\sqrt{1-x^2}}\,dx$

$= x\arcsin x + \frac{1}{2}\int u^{-1/2}\,du$

$= x\arcsin x + \frac{1}{2}\left(2u^{1/2}\right)$

$= x\arcsin x +$

Examples

d. $\int x^2 \sin x \, dx = uv - \int v\,du$

$u = x^2 \quad du = 2x\,dx$

$v = -\cos x \quad dv = \sin x\,dx$

$\cancel{u = \sin x \quad du = \cos x\,dx}$

$\cancel{v = \frac{1}{3}x^3 \quad dv = x^2\,dx}$

$u = 2x \quad du = 2\,dx$

$v = \sin x \quad dv = \cos x$

$-x^2\cos x + \int 2x\cos x\,dx$

$-x^2\cos x + 2x\sin x - 2\int \sin x\,dx$

$\boxed{-x^2\cos x + 2x\sin x + 2\cos x + C}$

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Examples

e. $\int \sec^3 x\,dx = \int \overset{dv}{(\sec^2 x)}\overset{u}{(\sec x)}\,dx$

$u = \sec x \quad du = \sec x \tan x$

$v = \tan x \quad du = \sec^2 x$

$\int \sec^3 x\,dx = \sec x \tan x - \int \sec x \underset{(\sec^2 x - 1)}{\tan^2} x\,dx$

$\int \sec^3 x\,dx = \sec x \tan x - \left(\int \sec^3 x - \sec x\,dx\right)$

$\int \sec^3 x\,dx = \sec x \tan x - \cancel{\int \sec^3 x\,dx} + \int \sec x\,dx$

$+\int \sec^3 x\,dx \qquad + \cancel{\int \sec^3 x\,dx}$

$\frac{\cancel{2}\int \sec^3 x\,dx}{\cancel{2}} = \sec x \tan x + \int \sec x\,dx$

$= \boxed{\frac{\sec x \tan x + \ln|\sec x - \tan x|}{2} + C}$

Further Tips/Guidelines

SUMMARY OF COMMON INTEGRALS USING INTEGRATION BY PARTS

1. For integrals of the form

$$\int x^n e^{ax}\, dx, \qquad \int x^n \sin ax\, dx, \qquad \text{or} \qquad \int x^n \cos ax\, dx$$

let $u = x^n$ and let $dv = e^{ax}\, dx$, $\sin ax\, dx$, or $\cos ax\, dx$.

2. For integrals of the form

$$\int x^n \ln x\, dx, \qquad \int x^n \arcsin ax\, dx, \qquad \text{or} \qquad \int x^n \arctan ax\, dx$$

let $u = \ln x$, $\arcsin ax$, or $\arctan ax$ and let $dv = x^n\, dx$.

3. For integrals of the form

$$\int e^{ax} \sin bx\, dx \qquad \text{or} \qquad \int e^{ax} \cos bx\, dx$$

let $u = \sin bx$ or $\cos bx$ and let $dv = e^{ax}\, dx$.

Using the Tabular Method

This can be used for integrals of the form $\int x^n \sin ax\, dx$, $\int x^n \cos ax\, dx$, $\int x^n e^{ax}\, dx$.

Ex. Find $\int x^2 \sin 4x\, dx$

Refer to pg. 530 Ex 7

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8.2 #5-10, 11-35 (e.o.o), 47-63 (e.o.o)