Lesson 7-3

Volume: Shell Method

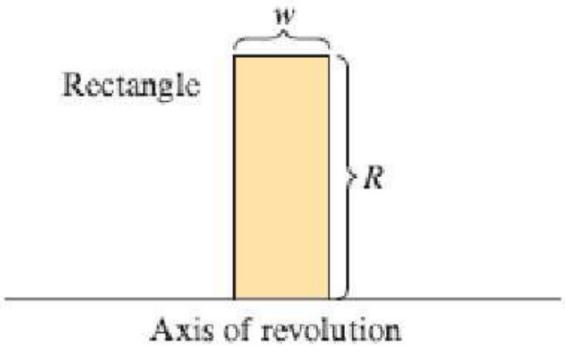
Objective

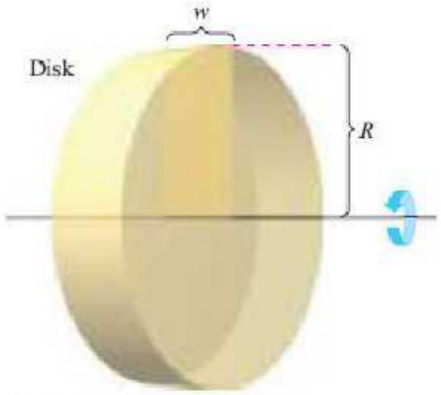
Students will…

- Be able to find the volume of a solid of revolution using the cylindrical method.

Volume

Recall that while area refers to two-dimensional shapes, while volume refers to three-dimensional shapes. An important application of definite integrals is finding the volume of a solid using its lateral area. The idea behind this method is that any three-dimensional shape could be viewed as being made up of multiple two-dimensional shapes.



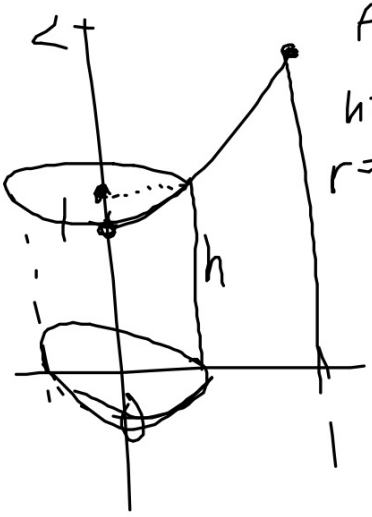


Volume of a disk: $\pi R^2 w$

Figure 7.13

Examples

Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^2 + 1$, $y = 0$, $x = 0$, and $x = 1$ about the y-axis, using the shell method.



$A = 2\pi rh$

$h = x^2 + 1$

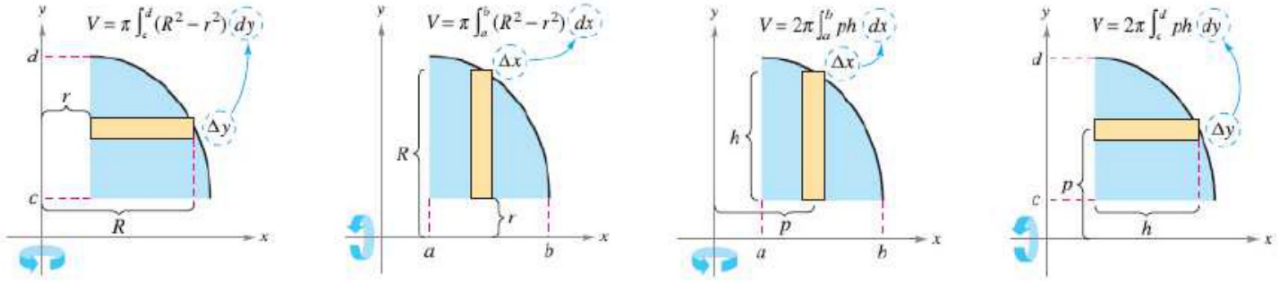
$r = x$

$$V = \int_0^1 2\pi x(x^2+1)\,dx = 2\pi \int_0^1 x(x^2+1)\,dx$$

$$= 2\pi \int_0^1 x^3 + x\,dx = 2\pi\left(\frac{1}{4}x^4 + \frac{1}{2}x^2\Big|_0^1\right)$$

$$= 2\pi\left(\frac{1}{4} + \frac{1}{2}\right) = \boxed{\frac{3}{2}\pi}$$

Comparison of Disk and Shell Methods



Disk method: Representative rectangle is perpendicular to the axis of revolution.

Shell method: Representative rectangle is parallel to the axis of revolution.

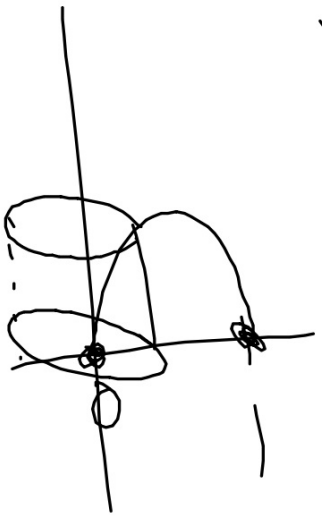
Figure 7.32

Disk

shell (cylindrical)

Examples

Find the volume of the solid of revolution formed by revolving the region bounded by $y = x - x^3$ and the x-axis $(0 \leq x \leq 1)$ about the y-axis.



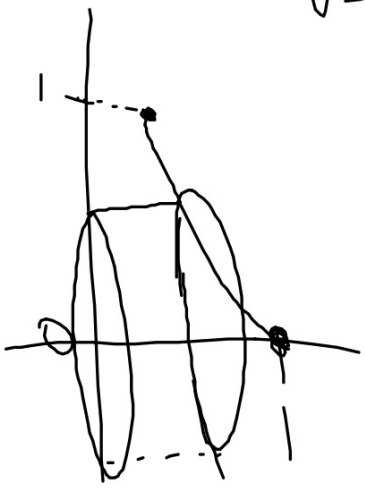
$$V = 2\pi \int_0^1 x\left(x - x^3\right) dx = 2\pi \int_0^1 x^2 - x^4 \, dx$$

$$= 2\pi \left(\frac{1}{3}x^3 - \frac{1}{5}x^5 \Big|_0^1 \right) = 2\pi \left(\frac{1}{3} - \frac{1}{5} \right) = \boxed{\frac{4}{15}\pi}$$

Examples

$u = -y^2$ $\quad du = -2y\,dy$ $\quad y\,dy = -\frac{1}{2}du.$

Find the volume of the solid of revolution formed by revolving the region bounded by the graph of $x = e^{-y^2}$ and the y-axis $(0 \le y \le 1)$ about the x-axis.



$$V = 2\pi \int_0^1 y\left(e^{-y^2}\right)dy = +\pi \int_{-1}^{0} e^u\,du$$

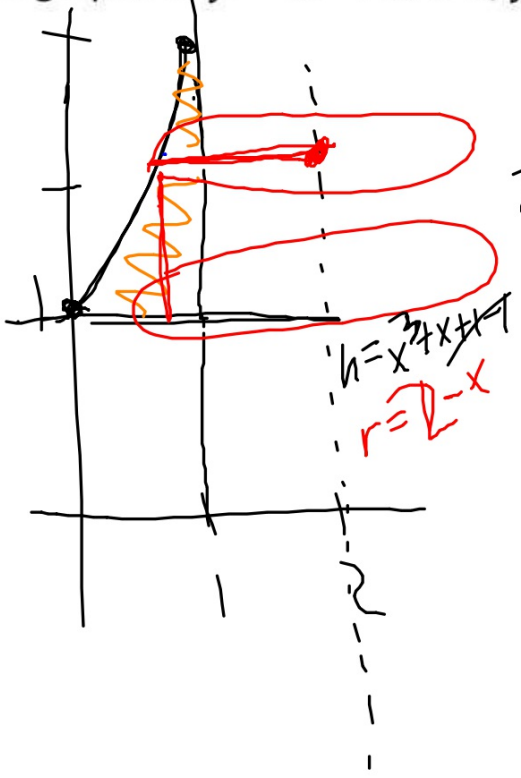
$$= \pi\left(e^u \Big|_{-1}^{0}\right) = \pi\left(1 - \frac{1}{e}\right)$$

$$= \boxed{\frac{e-1}{e}\pi}$$

$y-1=x^3+1$

Examples

Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^3 + x + 1, y = 1, x = 1$ about the line $x = 2$.



$$2\pi \int_0^1 (2-x)(x^3+x)\,dx$$

$$= 2\pi \int_0^1 2x^3+2x-x^4-x^2\,dx$$

$$= 2\pi \left(\frac{1}{2}x^4 + x^2 - \frac{1}{5}x^5 - \frac{1}{3}x^3 \Big|_0^1 \right)$$

$$= 2\pi \left(\frac{1}{2} + 1 - \frac{1}{5} - \frac{1}{3} \right) = \frac{29}{15}\pi$$

Homework 3/1

7.3 #1-4, 5-13 (odd), 15-18, 19-25 (odd), 29, 31