Warm Up 12/5

Evaluate the following trig functions without using a calculator.

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

1. $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

2. $\cos\frac{11\pi}{6} = \frac{\sqrt{3}}{2}$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

3. $\sec\frac{\pi}{4}$

$= \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\cancel{2}\boxed{\sqrt{2}}}{\cancel{2}}$

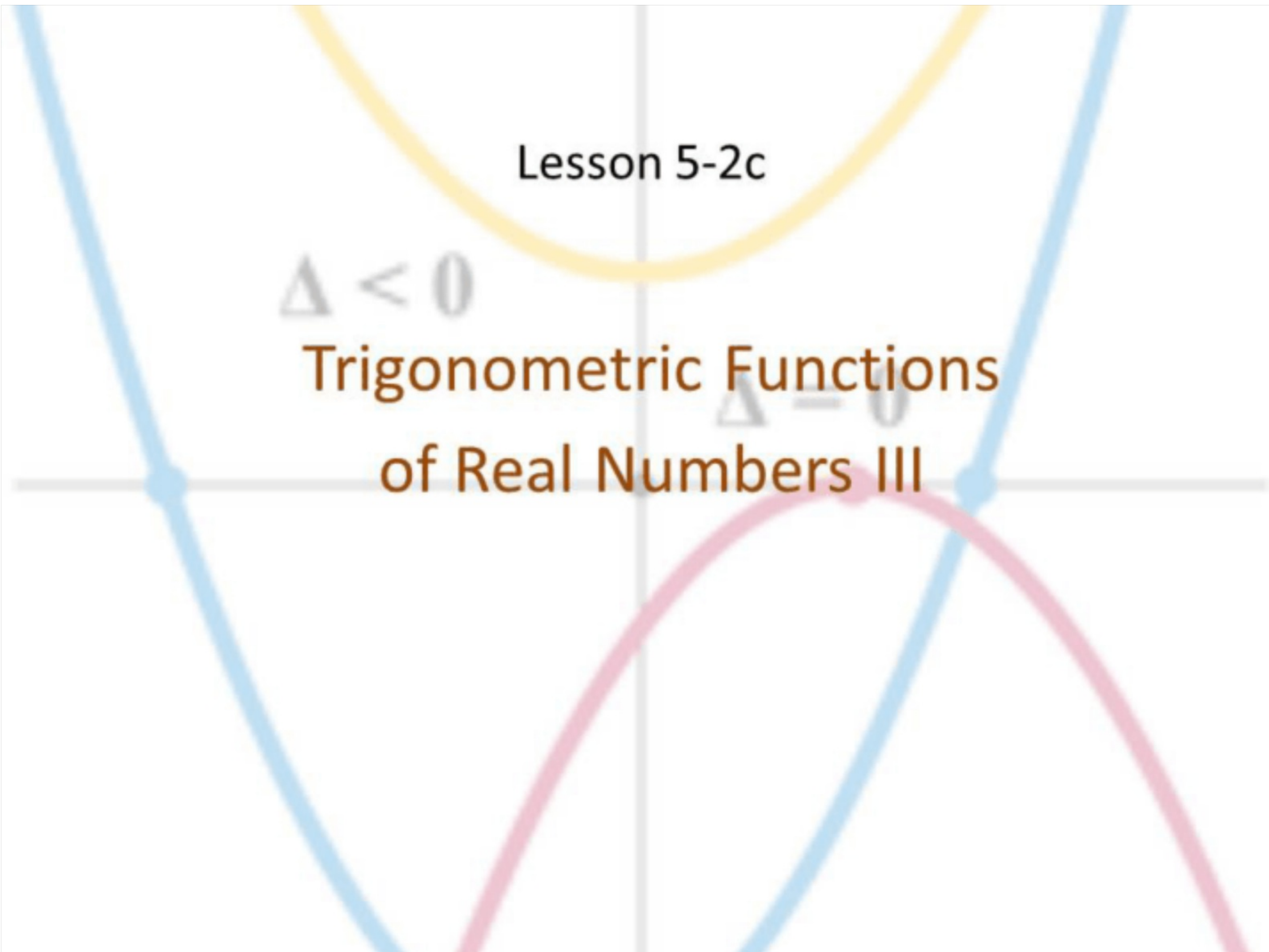
$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

4. $\tan\frac{4\pi}{3}$

$= -\frac{\sqrt{3}}{\cancel{2}} \cdot -\frac{\cancel{2}}{1} = \boxed{\sqrt{3}}$

Lesson 5-2c

Trigonometric Functions of Real Numbers III



Objective

Students will…

- Be able to rewrite trigonometric functions using the fundamental identities.
- Find all trigonometric functions from the value of one using the fundamental identities.

Soh Cah Toa

Mnemonic.

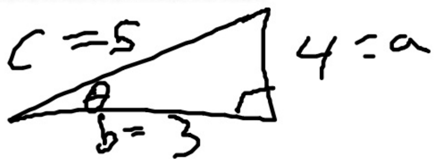
Recall that given a right triangle…

$$\sin t = \frac{opposite}{hypotenuse} \qquad \cos t = \frac{adjacent}{hypotenuse} \qquad \tan t = \frac{opposite}{adjacent}$$

$$\csc t = \frac{1}{\sin t} = \frac{hypotenuse}{opposite} \qquad \sec t = \frac{1}{\cos t} = \frac{hypotenuse}{adjacent}$$

$$\cot t = \frac{1}{\tan t} = \frac{adjacent}{opposite}$$

We can use the properties of right triangles to figure out the rest of the trigonometric functions.



$a^2 + b^2 = c^2$

$4^2 + b^2 = 5^2$

$b^2 = 9$

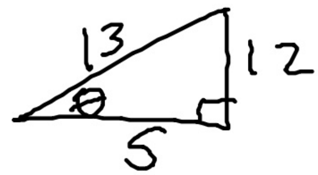
$b = 3$

$\sin t = -\frac{4}{5}$ opp / hyp $\cos t = \frac{3}{5}$ $\tan t = -4/3$

$\csc t = -\frac{5}{4}$ $\sec t = 5/3$

$\cot t = -3/4$

If $\cos t = -\frac{5}{13}$ and t is in quadrant II, find the values of all the trigonometric functions at t.



$\sin t = \frac{12}{13}$ $\quad \tan t = -\frac{12}{5}$ $\quad \csc t = \frac{13}{12}$

$\sec t = -\frac{13}{5}$ $\quad \cot t = -\frac{5}{12}$

Trigonometric Functions

Recall from the definitions of trigonometric functions that…

$$\tan t = \frac{\sin t}{\cos t} \qquad \cot t = \frac{1}{\tan t} = \frac{\cos t}{\sin t}$$

$$\sec t = \frac{1}{\cos t} \qquad \csc t = \frac{1}{\sin t}$$

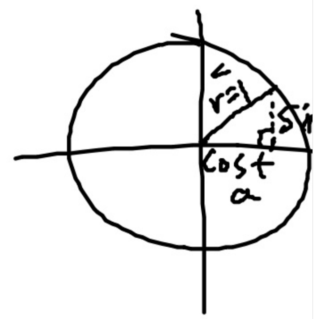
Coordinates on a Unit Circle

Now, also recall that on the unit circle, we defined the following:

$$\cos t = x \qquad \sin t = y \qquad \rightarrow \qquad (x, y) = (\cos t, \sin t)$$

Now, let's see how this can be applied on a unit circle.

Pythagorean Identities



Hence, we can now conclude the following identities:

Pythagorean Identities: (Note: $sin^2 t = (\sin t)^2$)

$sin^2 t + cos^2 t = 1$ $\qquad$ $tan^2 t + 1 = sec^2 t$ $\qquad$ $1 + cot^2 t = csc^2 t$

Also, moving some of these around using algebra:

$\sin t = \pm\sqrt{1 - cos^2 t}$ $\qquad$ $\cos t = \pm\sqrt{1 - sin^2 t}$

Rewriting Trigonometric Functions

We can also rewrite trigonometric functions using others.

Example: Write $\tan t$ in terms of $\cos t$, where t is in quadrant III.

$$\tan t = \frac{\sin t}{\cos t} = \boxed{\frac{\sqrt{1-\cos^2 t}}{\cos t}}$$

Examples

Write $\tan t$ in terms of $\sin t$, where t is in quadrant I.

$$\tan t = \frac{\sin t}{\sqrt{1-\sin^2 t}}$$

Write $\sec t$ in terms of $\tan t$, where t is in quadrant II.

$$\sec t = -\sqrt{\tan^2 t + 1}$$

Homework 12/5

TB pg. 417 #53-61 (odd), 63, 64

Application of Pythagorean Identities

Although these identities will be used much more extensively in the future, we can still make use of them here.

Example: If $\cos t = \frac{3}{5}$ and t is in quadrant IV, find the values of all other trigonometric functions at t.

We need to first find $\sin t$. We use our identity: $\sin t = \pm\sqrt{1 - cos^2 t}$

$$\sin t = \pm\sqrt{1 - cos^2 t} = \pm\sqrt{1 - \left(\frac{3}{5}\right)^2} = \pm\sqrt{1 - \left(\frac{9}{25}\right)} = \pm\sqrt{\frac{16}{25}} = \pm\frac{4}{5}$$

However, since we are told that t is quadrant IV, $\sin t = -\frac{4}{5}$